MODULE-1

OSCILLATIONS AND WAVES

Oscillations and vibrations play a more significant role in our lives than we realize. When you strike a bell, the metal vibrates, creating a sound wave. All musical instruments are based on some method to force the air around the instrument to oscillate. Oscillations from the swing of a pendulum in a clock to the vibrations of a quartz crystal are used as timing devices. When you heat a substance, some of the energy you supply goes into oscillations of the atoms. Most forms of wave motion involve the oscillatory motion of the substance through which the wave is moving. Despite the enormous variety of systems that oscillate, they have many features in common with the simple system of a mass on a spring. The harmonic oscillators have close analogy in many other fields; mechanical example of a weight on a spring, oscillations of charge flowing back and forth in an electrical circuit, vibrations of a tuning fork, vibrations of electrons in an atom generating light waves, oscillation of electrons in an antenna etc.,

PERIODIC MOTION

If we observe the motion of the pendulum of a clock, piston in a cylinder of an engine, we find that same motion along the same path is repeated again and again after equal intervals of time. Such a type of motion is called a periodic motion- defined as *a motion in which the body describes the same path in the same way again and again in equal intervals of time.*

SIMPLE HARMONIC MOTION

Simple harmonic motion is a particular case of periodic motion and is most fundamental type of periodic motion having single period. A Simple harmonic motion can be linear and angular. In general *we can define simple harmonic motion as a motion in which the acceleration of the body is directly proportional to its displacement from a fixed point and is always directed towards the fixed point.*

Characteristics of a Simple Harmonic Motion

1. The motion should be periodic.
2. When displaced from mean position, a restoring force, tending to bring it to the mean position and directed towards the mean position must act on the body.
3. The restoring force should be directly proportional to the displacement of the body from its mean position.

For a SHM Restoring force α – displacement

$$F = -k\,x$$

Here k is the proportionality constant known as spring constant. It represents the amount of restoring force produced per unit elongation and is a relative measure of stiffness of the material.

$$F_{Restoring} = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$Let\ \omega_o^2 = \frac{k}{m}$$

$$\frac{d^2x}{dt^2} + \omega_o^2 x = 0$$

Here ω_o is angular velocity = 2.π.f

f- is the natural frequency $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

The Solution is of the form $x(t) = A\cos\omega_o t + B\sin\omega_o t$

This can also be expressed as $x(t) = C\cos(\omega_o t-\theta)$ where $C = \sqrt{A^2+B^2}$ $\tan\theta = B/A$

MECHANICAL SIMPLE HARMONIC OSCILLATOR

We consider a mechanical spring which resists compression / elongation to be elastic. At the lower end of the spring, a body of mass m is attached. Mass of the spring is neglected .When the body is pulled down by a certain distance x and then released, it undergoes SHM. **When there are no external forces, the oscillations are said to be free oscillations.** The mass oscillates with its natural frequency

MASS SUSPENDED BY A SPRING (VERTICAL VIBRATIONS)

For a light spiral spring within the elastic limit the tension of the spring is proportional to the extension of the spring beyond its length that obeys Hooke's law.

Fig (1.1) shows a spring of length L suspended from a support at A. If a mass M is attached to its free end B, it will be stretched downward and its length will increase say by l that is BC=l. Due to the increase in the length of the spring the force exerted by the spring on the mass, according to Hooke's law, will be given by $-kl$, where k is the proportionality constant and depends upon the material and size of the spring and balances the tension T in the spring.

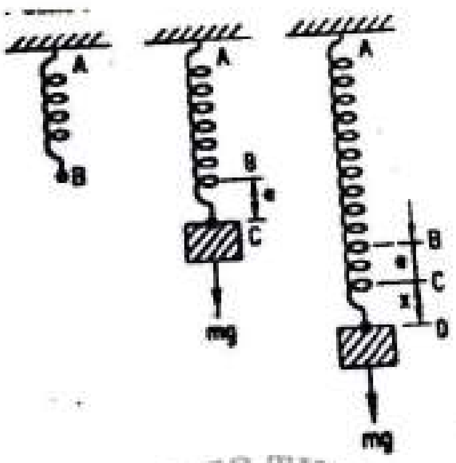


Fig. (1.1)

The constant *k* is known as spring constant or stiffness factor or simply force constant.

$T = mg = kl$ 1

Now if the load is displaced downward to a position D through a small distance x(CD=X), making the total increase in the length by (x+l), the tension in this position T' will be proportional to (x+l) i.e

$T' = k(x+l)$ 2

Therefore the resultant force acting on the mass will be T-T' or mg-T'. Hence we have resultant force on the mass.

$F = mg - T' = kl - k(l + x) or F = -kx$

Therefore acceleration of the mass = Force/ mass= -k/m 3

In the above equation k/m is constant hence the acceleration of the mass is proportional to its displacement and is directed towards its mean position C.

Thus the mass will execute SHM motion with C as the equilibrium position. From equation (3)

k/m = acceleration/x = acceleration per unit displacement.

Now the time period of SHM in seconds can be given by

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

Since $mg = kl$ we have $k = mg/l$ and substituting this value of k in equation (4) we have

$$T = 2\pi\sqrt{\frac{l}{g}}$$

VIBRATION OF SPRING MASS SYSTEM WHEN TWO OR MORE SPRINGS ARE USED

A) Springs in series

In case if the mass is connected to a spring which consists of two different springs of different stiffness factor, the time period can be calculated as below.

Let the stiffness factor of spring S_1 and S_2 be k_1 and k_2 respectively and he increase in the length of the spring S_1 be x_1 and that of S_2 be x_2. If x is the total increase in the length of the spring system because of mass m as shown in the figure (1.2).

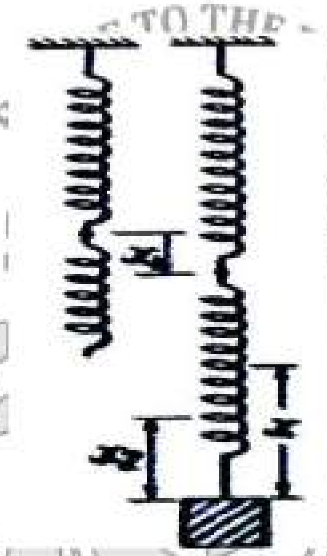


Fig. (1.2)

Now the same weight causes the elongation in each spring.

Therefore, $mg = k_1\, x_1$ and also $mg = k_2\, x_2$

Or $x_1 =$ — and $x_2 =$ —

If k_{eq} is the equivalent stiffness factor of the combination we have,

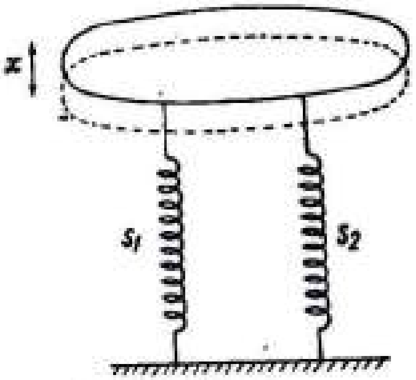
$Mg = k_{eq}\, X$ or $x =$ —

Substituting the values of x, x_1 and x_2 in equation (1) we have

— = — + — or — = — —

From equation (2) we can say in general, if a number of springs of different stiffness are connected in series the multi spring system can be regarded as consisting of single spring of equivalent stiffness factor.

B) Springs in parallel



Consider two springs S_1 and S_2 as shown in the figure connected in parallel. Each spring will share the total load and will have equal elongation say x. If k_1 and k_2 are stiffness factors for springs S_1 and S_2 and k be the equivalent stiffness factor for the combination.

Total restoring force = mg= x ……………………(1)

Restoring force in spring S_1= $k_1 x$ ……………………(2)

Restoring force in spring S_2= $k_2 x$ ……………………(3)

Now the total restoring force= Restoring force in spring S_1+ Restoring force in spring S_2

x x x

Or

x

Thus if a number of springs are connected in parallel the equivalent stiffness factor is the sum of individual stiffness factor.

FREE VIBRATIONS

IF we observe the vibrations of a simple pendulum, we find that the pendulum when let free vibrates with a fixed time period $= 2\pi\sqrt{\frac{l}{g}}$ or with a frequency $n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

This frequency is called the natural frequency of the pendulum. The pendulum whenever set to vibrate freely without any resistance; it will always vibrate with same frequency given by the above relation.

The frequency with which a body vibrates freely at its own is called its natural frequency.

Free vibrations can be defined as the vibrations in which the body vibrates with its own natural frequency when left free to itself.

DAMPED OSCILLATIONS: A body executing vibrations the amplitude keeps on decreasing because of frictional resistance to the motion and hence the vibrations die out after some time. The motion is said to be damped by friction and is called damped oscillations.

The resisting frictional force is proportional to the velocity of the body.

EXPRESSION FOR TIME PERIOD AND AMPLITUDE OF DAMPED VIBRATIONS

The damped system is subjected to

(i) A restoring force which is proportional to displacement but oppositely directed. This is written as -μy, where μ is a constant of proportionality or force constant.

(ii) A frictional force proportional to velocity but oppositely directed. This may be written as -r—, where r is the frictional force per unit velocity.

Since force = mass× acceleration = $m\frac{}{_2}$.

Therefore the equation of motion of the particle is given by

$$m\frac{}{_2} = -\mu y - r\frac{dy}{dt}$$

$$\text{or } \frac{}{_2} + \frac{}{} + \frac{}{}y = 0$$

$$\text{or } \frac{}{_2} + \quad \frac{}{} + \omega^2 y = 0 \text{ ....}(1)$$

where r/m = 2b and μ/m = ω^2

Equation (1) is a differential equation of second degree. Let its solution be

$$y = Ae^{\alpha t} \text{ ....}(2)$$

Where A and α are arbitrary constants.

Differentiating equation (2) with respect to 't' we get,

$$\frac{dy}{dt} = A\alpha e^{\alpha t} and \frac{d^2y}{dt^2} = A\alpha^2 e^{\alpha t}$$

Substituting these values in eq (1) we have

$$A\alpha^2 e^{\alpha t} + 2bA\alpha\, e^{\alpha t} + \omega^2 A e^{\alpha t} = 0$$

Or $Ae^{\alpha t}(\alpha^2+2b\alpha+\omega^2)=0$

As $Ae^{\alpha t} \neq 0$, therefore $\alpha^2+2b\alpha+\omega^2=0$

This gives $\alpha = -b \pm \sqrt{(b^2 - w^2)}\, t$

The general solution of eq(1) is given by

$$y = A_1 \exp[-b + \sqrt{(b^2 - w^2)}\, t + A_2 \exp[-b - \sqrt{(b^2 - w^2)}\, t \text{ ....}(3)$$

Where $A_1 and\ A_2$ are arbitrary constants

Depending upon the relative values of b and ω **following three cases are possible.**

Case 1: When $b^2 > w$

In this case $\sqrt{(b^2-w^2)}$ is real and less than b. Now the powers $-b+\sqrt{(b^2-w^2)}$ and $-b-\sqrt{(b^2-w^2)}$ are both negative. Thus the displacement y consists of two terms, both dying off exponentially to zero without performing any oscillations as shown in the figure. The rate of decrease of displacement is governed by the term $-b+\sqrt{(b^2-w^2)}\,t$ as the other term reduced to zero quickly relative to it. This type of motion is called *over-damped or dead beat.* This type of motion is shown by a pendulum moving in thick oil or by a dead beat moving coil galvanometer.

Case II: When b = w

If we put = w^2 in wquation (3), then this solution does not satisfy the eq (1). Let us consider that

$\sqrt{(b^2-w^2)}$ is not zero but this is equal to a very small quantity h i.e., $\sqrt{(b^2-w^2)}$ =h

Now eq. (3) reduces to $y = A_1 \exp[(-b+h)t + A_2 \exp[(-b-h)t$

$$= e^{-bt}\left[A_1 e^{ht} + A_2 e^{-ht}\right]$$

$$= e^{-bt}\left[A_1(1+ht+\ldots) + A_2(1-ht+\ldots)\right]$$

$$= e^{-bt}\left[(A_1+A_2) + ht(A_1-A_2)+\ldots\right]$$

$$= e^{-bt}\left[p+qt\right] \qquad \text{...(4)}$$

Where $p=(A_1+A_2)$ *and* $q=h(A_1-A_2)$

It is clear from eq. (4) that as t increases, the factor (p+qt) increases but the factor e^{-bt} decreases. In this case the particle tends to acquire its position of equilibrium much more rapidly than case I. Such a motion is called Critical damped motion. This type of motion is exhibited by many pointer instruments such as voltmeter, ammeter etc. in which the pointer moves to the correct position and comes to rest without any oscillation.

Case III: When $< w$

In this case $\sqrt{(b^2-w^2)}$ is imaginary. Let us write $\sqrt{(b^2-w^2)}$ = i $\sqrt{(w^2-b^2)}$ =iβ

Where β = $\sqrt{(w^2-b^2)}$ and i = $\sqrt{-1}$.

Then eq (3) becomes $y = A_1 \exp[(-b+i\beta)t + A_2 \exp[(-b-i\beta)t$

$$= e^{-bt}[A_1 e^{i\beta t} + A_2 e^{-i\beta t}]$$

$$= e^{-bt}[A_1(\cos\beta t + i\sin\beta t) + A_2(\cos\beta t - i\sin\beta t)]$$

$$= e^{-bt}[(A_1 + A_2)\cos\beta t + i(A_1 - A_2)\sin\beta t]$$

$$= e^{-bt}[a\sin\phi\cos\beta t + a\cos\phi\sin\beta t]$$

Where $a\sin\phi = (A_1 + A_2)$ *and* $a\cos\phi = i(A_1 - A_2)$

$$= e^{-bt}[a\sin(\beta t + \phi]$$

$$= ae^{-bt}\sin[\sqrt{(w^2 - b^2)}\,t + \phi] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

This equation represents the simple harmonic motion with amplitude ae^{-bt} and time period

$$T = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{(w^2 - b^2)}}$$

The amplitude of the motion is continuously decreasing owing to the factor e^{-bt} which is called the damping factor. Because of the value of $\sin[\sqrt{(w^2 - b^2)}\,t + \phi]$ varies between +1 and -1. Therefore the amplitude also varies between ae^{-bt} and $-ae^{-bt}$. The decay of the amplitude depends upon the damping coefficient b, it is called *under damped motion*.

QUALITY FACTOR

The quality factor is defined as 2π times the ratio of the energy stored in the system to the energy lost per period.

$$Q = 2\pi\frac{\text{energy stored in the system}}{\text{energy lost per period}} = 2\pi\frac{E}{PT}$$

Where **P** is the power dissipated and **T** is the periodic time

$$\therefore Q = 2\pi\frac{E}{(E/\tau)T} = \frac{2\pi\tau}{T} = \omega\tau \qquad (\because \boldsymbol{P = E/\tau})$$

$$\omega = (2\pi/T) = \text{angular frequency} \quad \text{where}$$

So it is clear that higher is the value of Q, higher would be the value of relaxation time τ i.e., lower damping.

In case of low damping $\omega = \omega_0$, so $Q = \omega\tau$ (constant of damped motion)

If k be the force constant and m, the mass of the vibrating system, then

$$\omega_0 = \sqrt{\frac{k}{m}} \quad and \quad \tau = 1/2b$$

$$\therefore Q = \frac{1}{2b} \times \sqrt{\frac{k}{m}}$$ This also shows that for low damping the quality factor is higher.

FORCED VIBRATIONS

So far we have discussed the vibrations in which the body vibrates at its own frequency without being subjected to any other external force. However, different situation arise when the body is subjected to an external force. Such vibrations are called *forced vibrations*.

Forced vibrations can be defined as the vibrations in which the body vibrates with a frequency other than its natural frequency under the action of an external periodic force.

The forces acting upon the particle are

1) a restoring force proportional to the displacement but oppositely directed, given by -μy, where μ is the force constant
2) A frictional force proportional to velocity but oppositely directed given by - — where r is the frictional force per unit velocity, and
3) The external periodic force, represented by F sinpt, where F is the maximum value of this force and $p/2\pi$ is its frequency.

So the total force acting on the particle is given by

-μy- — + F sinpt

By Newton's second law of motion this must be equal to the product of mass of the particle and its instantaneous acceleration i.e., $m\frac{d^2y}{dt^2}$, hence

$$m\frac{d^2y}{dt^2} = -\mu y - r\frac{dy}{dt} + F\sin pt$$

Or $$m\frac{d^2y}{dt^2} + \mu y + r\frac{dy}{dt} = F\sin pt$$

Or $$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{\mu}{m}y = \frac{F}{m}\sin pt$$

Or $$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = f\sin pt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where $\frac{r}{m} = 2b; \frac{\mu}{m} = \omega^2 \text{ and } \frac{F}{m} = f$

Eq. (1) is the differential equation of the motion of the particle.

In this case, when the steady state is set up, the particle vibrates with the frequency of applied force, and not with its own natural frequency. The solution of eq. (1) must be of the type

$y = A\sin(pt-\theta)$ ……………………………………(2)

Where A is the steady amplitude of vibrations and θ is the angle by which the displacement y lags.

Differentiating (2) we have

$$\frac{dy}{dt} = Ap\ \cos(pt-\theta)$$

$$\text{and } \frac{d^2y}{dt^2} = -Ap^2\sin(pt-\theta)$$

Substituting these values in eq(1) we get

$$-Ap^2\ \sin(pt-\theta) + 2bA\cos(pt-\theta) + \omega^2 A\sin(pt-\theta) = f\sin pt = f\sin\{(pt-\theta)+\theta\}$$
$$\text{or } A(\omega^2-p^2)\sin(pt-\theta) + 2bAp\cos(pt-\theta) = f\sin(pt-\theta)\cos\theta + f\cos(pt-\theta)\sin\theta$$

If this relation holds good for all values of t, the coefficients of $\sin(pt-\theta)$ and $\cos(pt-\theta)$ terms on both sides of this equation must be equal i.e., comparing the coefficients of $\sin(pt-\theta)$ and $\cos(pt-\theta)$ on both sides, we have

$A(\omega^2-p^2) = f\cos\theta$ ……………………….(3)
$\text{and } 2bAp = f\sin\theta$ …………………….(4)

Squaring equations (3) and (4) we get

$$A^2(\omega^2-p^2)^2 + 4b^2A^2p^2 = f^2$$
$$A^2\{(\omega^2-p^2)^2 + 4b^2p^2\} = f^2$$

$$A = \frac{f}{\sqrt{\{(\omega^2-p^2)^2 + 4b^2p^2\}}}$$

…………………………….(5)

While on dividing eq. (4) by eq. (3) we get

$$\tan\theta = \frac{2bAp}{a(\omega^2 - p^2)}$$

$$\theta = \tan^{-1}\left(\frac{2bp}{(\omega^2 - p^2)}\right) \quad ....(6)$$

Eq. (5) gives the amplitude of forced vibration and eq. (6) gives its phase.

Depending upon the relative values of p and ω, following three cases are possible

Case 1: When driving frequency is low i.e p<<ω. In this case, the amplitude of vibration is given by

$$A = \frac{f}{\sqrt{\{(\omega^2 - p^2)^2 + 4b^2 p^2\}}}$$

$$A = \frac{f}{\omega^2} \quad = \text{constant}$$

$$\text{And } \theta = \tan^{-1}\left(\frac{2bp}{(\omega^2 - p^2)}\right) = \tan^{-1}(0) = 0$$

This shows that the amplitude of vibration is independent of frequency of force. This amplitude depends on the magnitude of the applied force and force constant μ. The force and displacement are always in phase.

Case II: When p=ω i.e., the frequency of the force is equal to frequency of the body. In this case amplitude of vibration is given by

$$A = \frac{f}{2bp} = \frac{F}{r\omega} \quad \left(\because f = \frac{F}{m}, 2b = \frac{r}{m} \text{ and } p = \omega\right)$$

$$also\ \ \theta = \tan^{-1}\left(\frac{bp}{0}\right) = \tan^{-1}(\infty) = \pi/2$$

Thus the amplitude of vibration is governed by damping and for small damping forces, the amplitude of vibration is quite large. The displacement lags behind the force by $\pi/2$.

Case III: When p>>ω, the frequency of force is greater than the natural frequency ω of the body.

In this case

$$A=\frac{f}{\sqrt{p^2+4b^2p^2}}\approx\frac{f}{p^2}=\frac{F}{mp^2}$$

And $\theta=\tan^{-1}\left(\frac{2bp}{(\omega^2-p^2)}\right)=\tan^{-1}\left(-\frac{2b}{p}\right)\approx\tan^{-1}(0)=\pi$

Thus in this case, the amplitude A goes on decreasing and phase difference tends towards π .

RESONANCE

If we bring a vibrating tuning fork near another stationary tuning fork of the same natural frequency as that of vibrating tuning fork, we find that stationary tuning fork also starts vibrating. This phenomenon is known as resonance.

The phenomenon of making a body vibrate with its natural frequency under the influence of another vibrating body with the same frequency is called resonance.

Condition for amplitude resonance

In case of forced vibrations we know that

$$A=\frac{f}{\sqrt{\{(\omega^2-p^2)^2+4b^2p^2\}}}$$

For a particular value of p, amplitude becomes maximum. This phenomenon is known as amplitude maximum.

EXAMPLES OF RESONANCE

1. Oscillations of the stretched string kept under the influence of oscillating magnetic field caused by oscillating current. The string vibrates with maximum amplitude when the applied frequency matches with Natural frequency of the string.

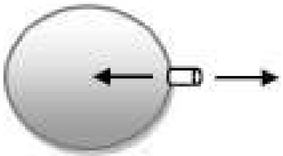
2. Sodium chloride crystal has alternately Sodium and Chloride ions. If an electric field is applied on the crystal, the charges would oscillate back and forth. The natural frequency is in Infrared range.

HEMHOLTZ RESONANTOR

It is used to analyze complex note. It consists of a hollow sphere of thin glass or brass with an opening through a narrow neck. It is filled with air. The opening receives exciting sound waves and the ears are kept close to the neck. When air is pushed into the sphere and released, the pressure will drive it out. The volume of air in the container behaves as a mass on a spring which is pulled down and released. Compressed air tends to move out and creates low pressure inside. The air will oscillate into and out of the container at its natural frequency given by the expression

$$f = \frac{V}{2\pi}\sqrt{\frac{A}{lv}}$$

V is the velocity of sound, l the length, A the area of the opening, v the volume of the resonator



SHOCK WAVES

MACH NUMBER

In aerodynamics the speeds of bodies moving in a fluid medium are classified into different categories on the basis of Mach number.

It is defined as "The ratio of the speed of the object to the speed of sound in the given medium".

i.e. .Mach number = ―――――――――――

M= -

Where, M= Mach number

V= object speed

a = Speed of sound in the medium

Note* It does not have any unit. It is a pure number.

DISTINCTION BETWEEN ACOUSTIC, ULTRASONIC, SUBSONIC AND SUPERSONIC WAVES

Acoustic waves: These are longitudinal waves that travel in a medium with the speed of sound in that medium .These waves can propagate in solids, liquids and gases.

Acoustic waves can be classified into the following three types depending upon their frequency.

1) Infrasonic: These acoustic waves have frequency less than 20Hz. The human ear cannot detect these waves.
2) Audible: These acoustic waves have frequency between 20Hz to 20K Hz. The human ear is sensitive to these waves. They cause sensation of hearing in human beings.
3) Ultrasonic: These acoustic waves have frequency more than 20K Hz. The human ear is not sensitive to these waves.

*An acoustic wave is sound wave.

* It moves with a speed of 333m/s in air at STP
*Sound waves have frequencies between 20 Hz to 20K Hz.
* Amplitude of Acoustic wave is very small.

Ultrasonic waves: Ultrasonic waves are pressure waves having frequencies beyond 20K Hz. But they travel with the same speed as that of sound .The Mach number =1

- Amplitude of ultrasonic wave is also small.

Subsonic wave: If the speed of mechanical wave or body moving in the fluid is lesser than that of sound, such a speed is referred to as subsonic wave. The Mach number <1.

Ex: The vehicles such as motor cars, trains, flight of birds is also subsonic.

- For a body moving with subsonic speed, the sound emitted by it manages to move ahead and away from the body since it is faster than the body.

Supersonic waves: Supersonic waves are mechanical waves which travel with speeds greater than that of sound. i.e., with speed for which, Mach number >1

A body with supersonic speed zooms ahead by piercing its own sound curtain, leaving behind a series expanding sound waves with their centers displaced continuously along its trajectory. Today's fighter planes can fly with supersonic speed .Amplitude of supersonic waves will be high it affects the medium in which it is travelling.

SHOCK WAVES

- Any fluid that propagates at supersonic speeds, gives rise to a shock wave.
- Shock waves are produced in nature during earth quakes (as seismic waves which travel with speeds ranging from 2km/s to 8km/s).
- When lightning strikes.

  Shock waves can be produced by a sudden dissipation of mechanical energy in a medium enclosed in a small space.

  "A shock wave is a surface that manifests as a discontinuity in a fluid medium in which it is propagating with supersonic speed".

  They are characterized by sudden increase in pressure, temperature and density of the gas. Through which it propagates.

  Shock waves are as strong or weak depending on the magnitude of the instantaneous change in pressure and temperature in the medium

  Ex: The shock waves created by the explosion of crackers, bursting of an automobile tire, during lighting thunder, during nuclear explosion.
- For strong shock waves Mach no. is high.
- For weaker shock waves the Mach no. is low(lesser than 1)

APPLICATIONS OF SHOCK WAVES

- ➢ They are used in the treatment of kidney stones.
- ➢ Cell information

- Wood preservation
- Use in pencil industry
- Gas dynamic studies
- Shock waves assisted needle less drug delivery
- Treatment of dry bore wells.

BASIC LAWS OF CONSERVATION OF MASS, MOMENTUM AND ENERGY

Conservation means the maintenance of certain quantities unchanged during physical process.

- Conservation laws apply to closed systems.
- A closed system is the one that does not exchange any matter with the outside and is not acted on by outside forces.

The conservation of mass, momentum and energy are the three fundamentals principles of classical physics.

1) ***Law of conservation of mass:***

"The total mass of any isolated system remains unchanged and is independent of any chemical and physical changes that could occur within the system"

Or

"The total mass of a system remains constant as mass can neither be created nor destroyed"

2) ***Law of conservation of momentum:***

"In a closed system the total momentum remains a constant"

Or

"When two objects collides in an isolated system, the total momentum of the objects before collision =the total momentum of the objects after collision"

3) ***Law of conservation of Energy:***

"The total energy of a closed system remains constant and is independent of any changes occurring within the system"

TYPES OF SHOCK WAVES

- ***Stationary shock wave:*** In a stationary shock wave, the shock front remains stationary with respect to fixed observer.
- ***Moving shock wave:*** such shock waves travel in fluids which are either stationary or move with small speeds with respect to the observer.
- ***Normal shock wave:*** if the shock wave is perpendicular to the flow of gas, it is known as normal shock wave.
- ***Oblique shock wave:*** if the shock wave is at some angle (other than 90^0) to the flow of gas it is known as oblique shock waves.

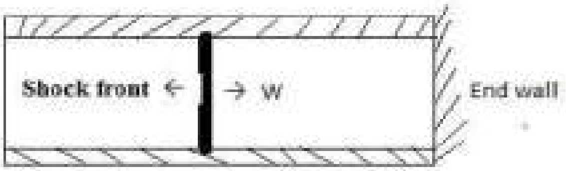
DERIVATION OF NORMAL SHOCK RELATIONSHIP USING SIMPLE BASIC CONSERVATION EQUATIONS (RANKINE-HUGONIT EQUATIONS)

Consider a shock wave propagating with a speed W in a shock tube.

The conditions of the shock wave at the downstream end can be determined by solving the equations for conservation of mass, momentum and energy, as applied to the shock region. These equations are known as Rankine-Hugonit relations.

* Consider two regions with reference to the shock front, one which is head of the shock front and the other behind it.

*Both the regions are at far enough distance from the shock front so that equilibrium conditions prevail the two regions where, the physical conditions such as pressure, density etc. are uniform.



Let us consider pressure (P_1) temperature (T_1), density (ρ_1), and enthalpy (h_1) before creation of the shock wave.

Similarly; (P_2), (T_2), (ρ_2) and (h_2) be the corresponding values, after creation of the shock wave.

Then, the three conservation relations, which correspond to the conservation laws.

$u_1 = \rho_2 u_2$ ----------- (1) conservation of mass

Where u_1 is the velocity of the fluid ahead of the shock.
u_2 is the velocity of the fluid following shock

$P_1 + \rho u_1^2 = P_2 + \quad u_2^2$ ----------- (2) conservation momentum

And $h_1 + u_1^2/2 = h_2 + u_2^2/2$ ------- (3) conservation of energy

These 3 equations along with the equation of states

$PV = RT$------ (4)

Lie at the roots of all the aerodynamic problems and applications. Using the above equations, the following normal shock relations called Rankine-Hugonit equations can be derived in which P_5 and T_5 represent pressure and temperature at the downstream and behind the reflected shock wave.

1) $- = [\quad - \quad]$

2) $- = - \left[\frac{(-)+-}{+(-)-}\right]$

3) $- = \frac{-}{-}$

4) $- = - \left[\frac{(-)+-}{+(-)-}\right]$

METHODS OF CREATING SHOCK WAVES IN THE LABORATORY USING A SHOCK TUBE:

Shock waves can be created in the laboratory by

(a) Using a Reddy shock tube
(b) Detonation
(c) Very high pressure gas cylinder
(d) Combustion
(e) Using small charge of explosives.

CHARACTERISTICS OF SHOCK TUBE

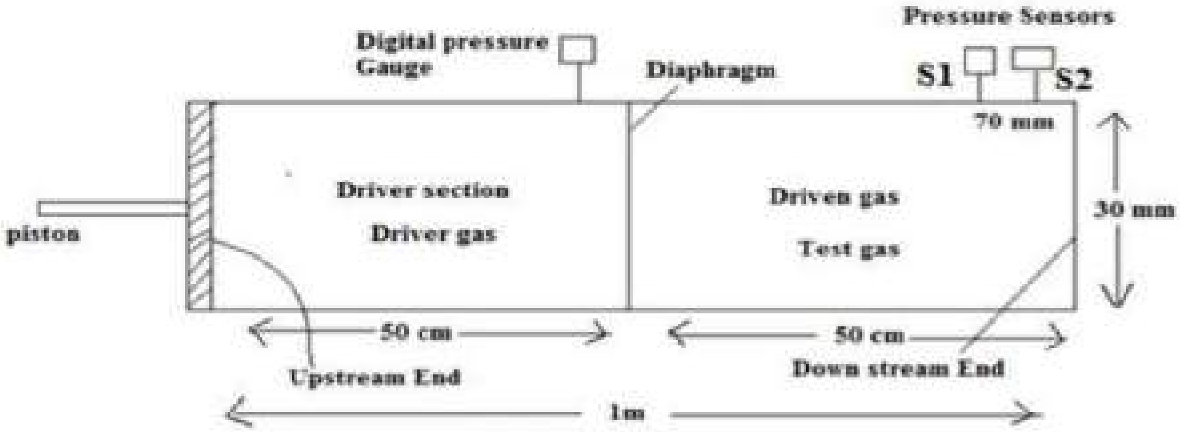
- The Reddy tube operates on the principle of free pision driven shock tube (FPST)
- It is a hand operated shock producing device.
- It is capable of producing Mach no. exceeding 1.5.
- The rupture pressure is a function of the thickness of the diaphragm.
- Temperature exceeding 900K can be easily obtained by the Reddy tube by using helium as the driver gas and argon as the driven gas. This temperature is useful in the chemical kinetic studies.

DESCRIPTION OF HAND OPERATED REDDY SHOCK TUBE

Reddy tube is a hand operated shock tube capable of producing shock waves by using human energy.

It is a long cylindrical tube with two separated by a diaphragm.

Its one end is fitted with a piston and the other end is closed.



Description:

- Reddy tube consists of a cylindrical stainless steel tube of about 30mm diameter and length 1m.
- It is divided into two sections each of length 50cm.
- One is the driver tube and other one is the driven tube.
- The sections are separated by a 0.1mm thick aluminium or paper diaphragm.
- A piston is fitted at the far end of the driver section
- A digital pressure gauge is mounted in the driver section next to the diaphragm.
- Two piezoelectric sensors S_1 and S_2 are mounted 70mm apart towards the closed end of the shock tube.
- A port is provided at the closed end of the driven section for filling the test gas to the required pressure.
- The driver section is filled with a gas termed as the driver gas which is held at a relatively high pressure due to the compressing action of the piston .the gas in the driven section is termed as driven gas or test gas.

Working:

- The driver gas is compressed by pushing the piston hard into the driver tube until the diaphragm ruptures
- The driver gas rushes into the driven section and pushes the driven gas towards the far downstream end. This generates a moving shock wave that traverses the length of the driven section
- The shockwave instantaneously raises the temperature and pressure of the driven gas as the shock moves over it.
- The propagating primary shock wave is reflected from the downstream end. After the reflection, the test gas undergoes compression which boosts its temperature and pressure to still higher values by the reflected shockwaves. This state of high values of pressure and temperature is sustained at the downstream end until an expansion wave reflected from the upstream end of the driver tube arrives there and neutralizes the compressions partially expansion waves are created at the instant the diaphragm is ruptured and they travel in a direction opposite to that of the shock wave.

- The period over which the extreme temperature and pressure conditions at the downstream end is sustained, is typically in the order of millisecond, however , the actual duration depends on the properties of the driver and test gases and dimension of the shock tube.
- The pressure rise caused by the primary shock waves and also the reflected shock wave are sensed as signals by the sensors S_1 and S_2 respectively, and they are recorded in a digital cathode ray oscilloscope.
- Since the experiment involve typically, millisecond duration measurements, the rise time of the oscilloscope should be a few microsecond. Hence an oscilloscope with a band width of 1MHz or more is required. From the recording in the CRO, the shock arrival times are found out by the associated time base calculations, using the data so obtained, Mach number, pressure and temperature can be calculated.

MODULE-2

ELASTIC PROPERTIES OF MATERIALS

ELASTICITY: Property by virtue of which materials regain their original shape and size after the removal of deforming forces.

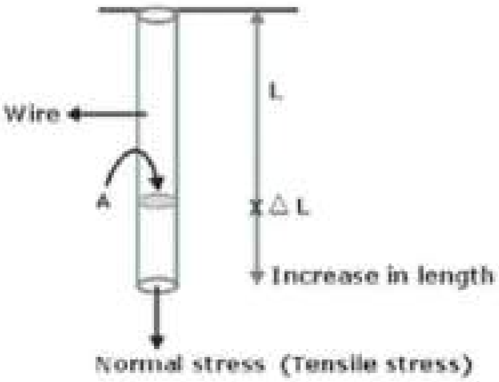
STRESS AND STRAIN: When a body is subjected to deforming force, a restoring force is developed in the body. The restoring force per unit area is known as stress (T). This restoring force is equal in magnitude but opposite in direction to the applied force. If F is the force applied and A is the area of cross section of the body, then $T = F/A$.

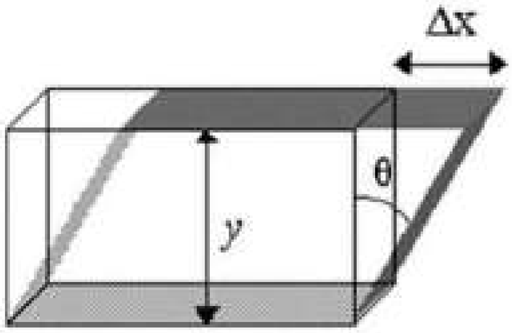
The SI unit of stress is Nm^{-2} or pascal (Pa) and its dimensional formula is $[ML^{-1}T^{-2}]$.

The ratio of change in dimension (ΔD) of a body to the original dimension (D) is called strain.
Strain has no units or dimensions. **Strain = $\Delta D/ D$**

TYPES OF STRESSES AND STRAINS

Normal stress: If a force is applied normal to the cross section of the body, then the restoring force developed per unit area is called normal or longitudinal stress. Longitudinal stress may be of two types i.e., tensile stress or compressive stress i.e. it can cause either extension (tensile stress) or compression (compressive stress) depending on the direction. In both cases, there is change in length of the body.





Shearing Stress: If a deforming force is applied parallel to the surface area of the body, there is relative displacement between opposite faces. The restoring force per unit area developed due to applied tangential force is known as tangential or shearing stress.

Longitudinal strain: The ratio of change in the length (ΔL) to the original length (L) of the body is known as longitudinal strain.
Longitudinal $strain = \Delta L/ L$

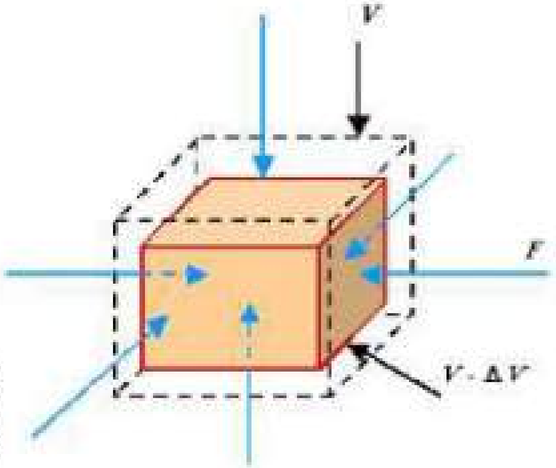
Shearing strain: As a result of applied tangential force, there **is relative displacement** Δx between opposite faces of the body as shown in figure. The strain so produced is known as shearing strain.

Shearing strain is defined as the ratio of relative displacement of the faces Δx to the height 'h'.

i.e., Shearing strain= $\tan\theta = \Delta x$ /h

where, θ is the angular displacement of the surface from the vertical (original position). Usually θ is very small and hence $\tan\theta \approx \theta = \Delta x$ /h

Volumetric strain: If forces are applied normal to the surface of a body in all directions it undergoes change in volume. The ratio of change in volume (ΔV) to the original volume (V), without any change in shape is called volumetric strain and stress causing is called normal stress.



HOOKE'S LAW

It states that Stress is directly proportional to strain within the elastic limit i.e. **stress α strain or**

Stress / strain = constant,

is called the modulus of elasticity or elastic constant. It is a characteristic property of the material.

YOUNG'S MODULUS (Y)

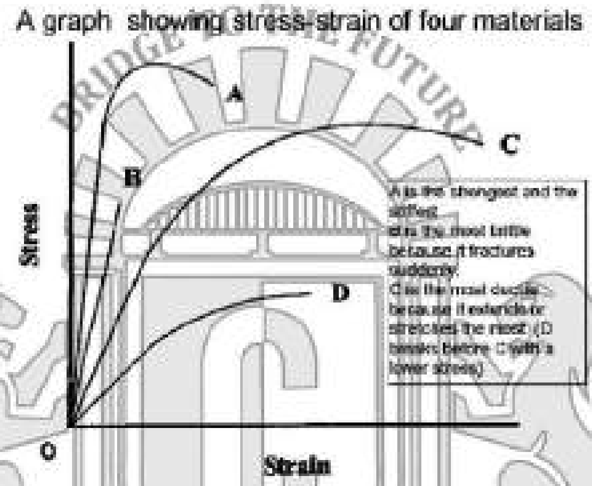
Young's Modulus is defined as the ratio of longitudinal (linear) stress to longitudinal strain with in the elastic limit. **Since strain is dimensionless quantity, the unit of Young's Modulus is the** same as that of stress i.e. Nm^{-2}

Y = —— = —

Where, F represents the force applied normal to the area 'A' of a wire of length 'L' and 'l' is the change in length.

For metals like iron, steel, copper, aluminum **etc., the Young's moduli are very large.** Therefore, these materials require a large force to produce small change in length.

STRESS-STRAIN CURVE



FACTORS AFFECTING ELASTICITY

Material Body generally can be single crystalline or polycrystalline and correspondingly behave elastically different. Single crystals when subjected to deformation show a remarkable increase in their hardness.

EFFECT OF HAMMERING AND ROLLING

Hammering and rolling helps in breaking crystal grains into smaller units, which result in an increase of their elastic properties. The slipping, (sliding between cleavage planes) starting at a weak spot proceeds within the grain and stops at its boundary with the adjoining crystal. No further flow possible makes that material hard.

EFFECT OF ANNEALING

Annealing (heating and cooling gradually) favors grain growth in a particular crystallographic direction results in large grains or less grain boundaries in the body, generally result in a decrease in their elastic properties or an increase in the softness (plasticity) of the material. Metals with smaller grains are stronger than those with larger grains.

Sir Lawrence Bragg said ***"in order to be strong, a metal must be weak"***

EFFECT OF IMPURITIES

Suitable impurities deliberately added can alter elastic properties of metals as they settle between the grains and brings connectivity between two grains.

Increase or decrease in elastic properties depends on the elastic or plastic property of the impurity added respectively. Carbon is added to molten iron to increase the elasticity of iron.

EFFECT OF TEMPERATURE

Increase in temperature generally decreases the elastic property.

Elastic properties of Invar steel are not affected by any change in temperature.

IMPORTANT USEFUL TERMS

Ductility: It is the property by virtue of which a material can acquire a large permanent deformation without any fracture or rupture.

Brittleness: Property by virtue of which a material gets ruptured without any appreciable amount of permanent deformation being produced in it.

Plasticity: Is the ability of a material to get deformed continuously and permanently without any rupture.

Elastic after effect: Delay in recovering back the original condition after the removal of deforming forces within elastic limit. (glass like amorphous materials show slow recovery whereas metals , quartz like crystalline materials show fast)

Elastic fatigue: loss of materials strength when subjected to repeated strain.

Working stress and Factor of safety: Generally working stress will be set to far below UTS within the elastic limit for the safety purpose.

Factor of safety = tensile strength or breaking stress/ working stress.(generally 5-10)

RIGIDITY MODULUS (η): The ratio of shearing stress to the corresponding shearing strain is called the shear modulus or Rigidity modulus of the material.

$$\eta = \frac{F/A}{\theta} \text{ Nm}^{-2}$$

F is the tangential force and θ is the shear strain.

BULK MODULUS (*K*): Bulk Modulus is defined as the ratio of normal stress to volumetric strain with in the elastic limit.

$$K = \frac{}{v}$$

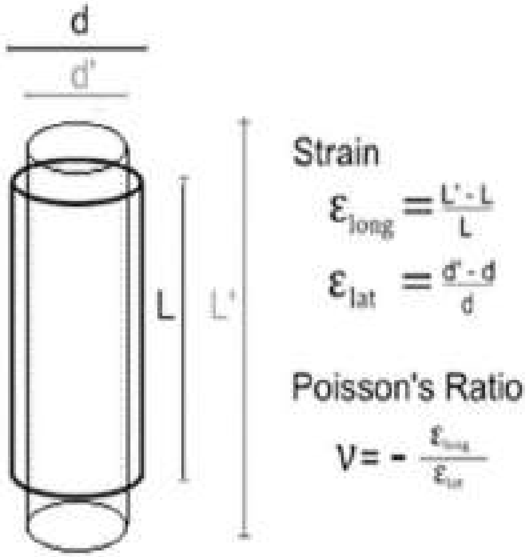
F/A is the normal stress, v is the change in volume and V is the original volume.

K is also referred as incompressibility and compressibility represents strain per unit stress.
α is increase in length per unit length per unit stress in the direction of the stress and β is decrease in length per unit length per unit stress in a direction perpendicular to the stress.

β is **lateral strain per unit stress** and **α** is the **longitudinal strain per unit stress**.

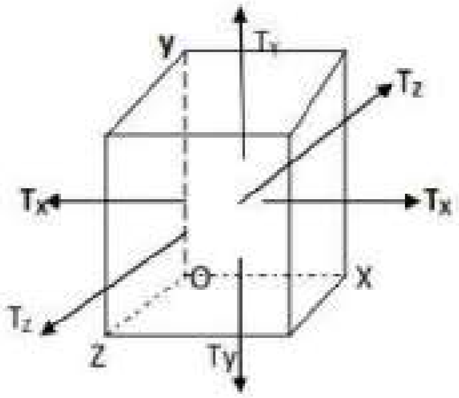
POISSON'S RATIO (σ) is the ratio of lateral strain to the longitudinal strain

$$\sigma = \beta / \alpha$$



EXPRESSION FOR BULK MODULUS K IN TERMS OF α AND β

Bulk modulus is the ratio of normal stress to volume strain.



Consider a unit cube (OX = OY =OZ = 1). The initial volume of the unit cube = V = 1
Let the cube be subjected to tensile stresses Tx, T_Y, T_Z along the X,Y and Z axes. Each of these stresses is tensile along the direction in which it is applied and compressive in directions perpendicular to it. T_X produces extension of the side OX and T_Y and T_Z produce compression of OX.

Let α represents the linear strain per unit stress and β is the lateral strain per unit stress.

Due to these stresses the dimensions of OX, OY and OZ are altered and can be written as

$OX = 1+\alpha T_x - \beta T_y - \beta T_z$ {linear Strain/T = α }

$OY = 1+\alpha T_y - \beta T_z - \beta T_x$

$OZ = 1+\alpha T_z - \beta T_x - \beta T_y$

Final volume = $\{1+ (\alpha-2\beta)(T_x+ T_y+ T_z)\}$ (Neglecting higher order terms of α and β)

Using $T_x= T_y= T_z = T$

Final volume = $\{1+ (\alpha-2\beta)(3T)\}$

Volume strain = Final volume – Initial Volume = $\{(\alpha-2\beta)(3T)\}$

Bulk Modulus 'K' = ———

K= ———

Expression for Young's modulus

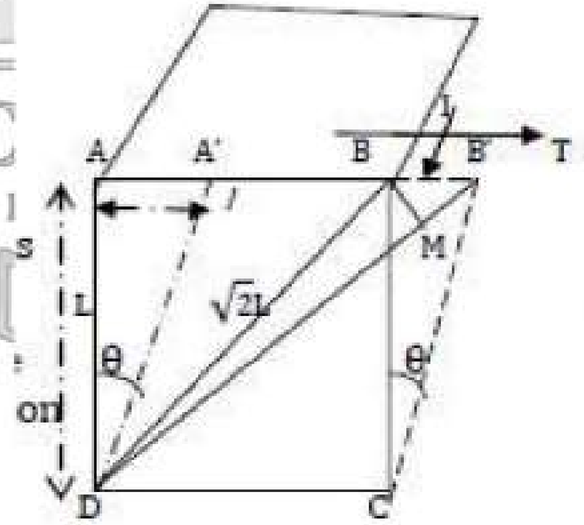
Y = Linear stress/ linear strain

$(F/A)/(l/L) = T/T\alpha = 1/\alpha$ (α is strain/T

$Y = 1 / \alpha$

EXPRESSION FOR RIGIDITY MODULUS

Consider a cube of dimension 'L. The bottom face of the cube is fixed and a tangential stress 'T' is applied along the top face.



Under the action of the tangential force, the cube gets deformed to A'B' C D. The shear strain is equal to θ and is given by the ratio, $\theta = BB'/BC = l/L$

The tangential stress is equal to a tensile stress T along BD and compressive stress T along AC.

These tensile and compressive stresses produce extension of the diagonal BD.

The increase in length of diagonal BD = MB'

If α is linear strain per unit stress and β is the lateral strain per unit stress,

The strain produced along the diagonal BD = T(α+ β) = MB' /BD

MB' = BB' cos (BB'M) = BB' cos 45^0 = $l/\sqrt{2}$

Therefore T (α + β) = —— = — = –

Or – = ——— = n

N= ———

RELATION BETWEEN Y, N AND K

We know that K = ———

= ———

= ———

$1-2\sigma$ = — ----------------(i)

n = ——— = ——— = ———

$2+2\sigma = Y/n$ ----------------(ii)

Add (i) and (ii) and rearrange the terms to obtain the relation between the elastic modulii as

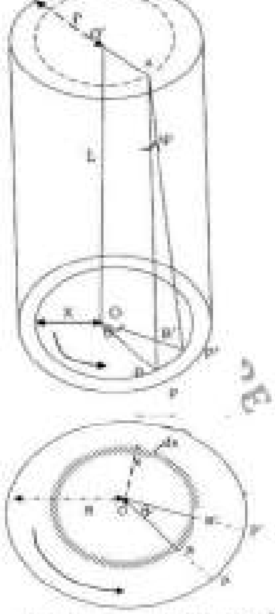
— — $\frac{}{n}$

THEORETICAL LIMITS OF σ

The elastic moduli are positive, so $(1-2\sigma)$ is positive and it places a upper limit equal to 0.5 on the value of σ. Similarly we can deduce the lower limit to be equal to -1 from eqn. (ii).

TORSION OF A CYLINDER

Consider a cylinder of length 'L' and radius 'R'. Let the cylinder be clamped at the upper end and a twisting couple be applied at the lower end. Let θ be the angle of twist.



The cylinder can be considered as made up of a number of co-axial hollow cylinders of varying radii (0 to R). At the bottom end, each of these radii are twisted through an angle, θ due to the applied external couple. As a result, OB is displaced to OB' and OP is displaced to OP' without change in dimensions.

A line AB parallel to OO' is displaced to AB' through an angle, φ, called the angle of shear. This is an example of pure shear as there is no change in either length or radius of the cylinder.

We can calculate the twisting couple on the co-axial cylinder of radius OB (OB = x) and integrate the expression between the limits, x=0 and x=R to obtain the magnitude of the twisting couple on the cylinder.

We can write from the geometry of the figure,

$BB' = x\,\theta = L\varphi$

Or, $\varphi = x\,\theta / L$ ---------------(i)

If δ F is the tangential force acting on a cylindrical shell of radius 'x' and thickness, ' dx ',

The tangential stress = δF /area of the shell = δF /2πx dx ---------(ii)

The rigidity modulus = tangential stress/shear strain

n = δF /2πx dx) /φ ------(iii)

δF = 2π n x dx. φ ---------(iv)

Twisting couple on the hollow cylinder of radius 'x' and thickness, 'dx' can be written as

$\boldsymbol{\delta C = tangential\ force \times distance = n.\ \varphi.\ 2\pi x\ dx.\ x}$ -----------(v)

n , is the rigidity modulus of the material and is given by the ratio of tangential stress to shear strain.

Substitute for φ in eqn. ii to obtain

$\boldsymbol{\delta C = 2\pi\ n\ \theta\ x2dx\ .\ x\ L = 2\pi n\theta\ x^3\ dx\ L}$ ---------------------------------(vi)

Twisting couple on the solid cylinder of radius 'R',

$$C = \frac{2\pi n\theta}{L}\int_0^R x^3\,dx = \frac{2\pi n\theta}{L}\,[R^4/4] = \frac{\pi n R^4\theta}{2L}$$

TORSIONAL PENDULUM

A rigid wire of length' L' and radius 'r' is fixed at one end and forms the axis of rotation for a regular or an irregular body attached to the free end. When the wire is twisted at the free end, the body is set into oscillations and these oscillations are called torsional oscillations.

Let I be the moment of inertia of the body about the given axis and T be the time period for torsional oscillations. Let the restoring couple per unit twist be 'C' and 'θ' be the angle of twist.

Angular acceleration = -(C /I) θ

$$I\,\frac{d^2\theta}{dt^2} = -C\,\theta$$
$$\frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$$

represents the simple harmonic equation for torsional oscillations.

From the above equation, we can get the time period for torsional oscillations

$$T = 2\pi\sqrt{I/C}$$

$$\text{Or, } \frac{I}{T^2} = \frac{C}{4\pi^2} \qquad \text{where } C = \frac{\pi n r^4 \theta}{2L}$$

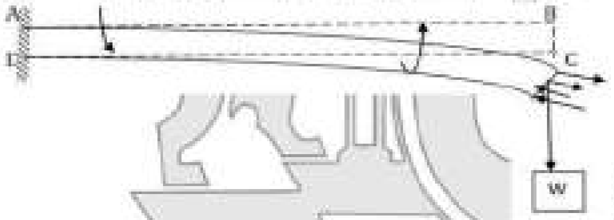
Period = 2π √ (displacement/acceleration)

For a given wire, C is constant and hence the ratio I/T^2 are constant irrespective of the body or the axis of rotation. This principle is used to determine the moment of inertia of the irregular objects and rigidity modulus of the material wire using $n = (8\pi l/r^4)(I/T^2)$

BENDING MOMENT OF A BEAM

A beam is a structural member whose length is very large compared to other dimensions. In the simple theory of bending of beams, the shear stresses are neglected and only tensile or compressive stresses are considered.

Consider a beam fixed at one end and loaded at the other end. Due to the applied force, a force of reaction is set up at the fixed end of the beam. These two equal and opposite forces constitute a couple which tends to rotate the beam. Since the beam is elastic a restoring couple is set up to bring the beam back to equilibrium.

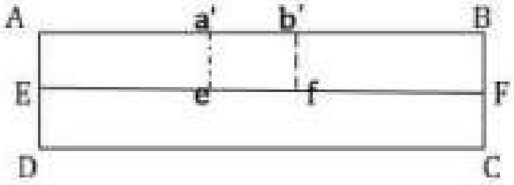


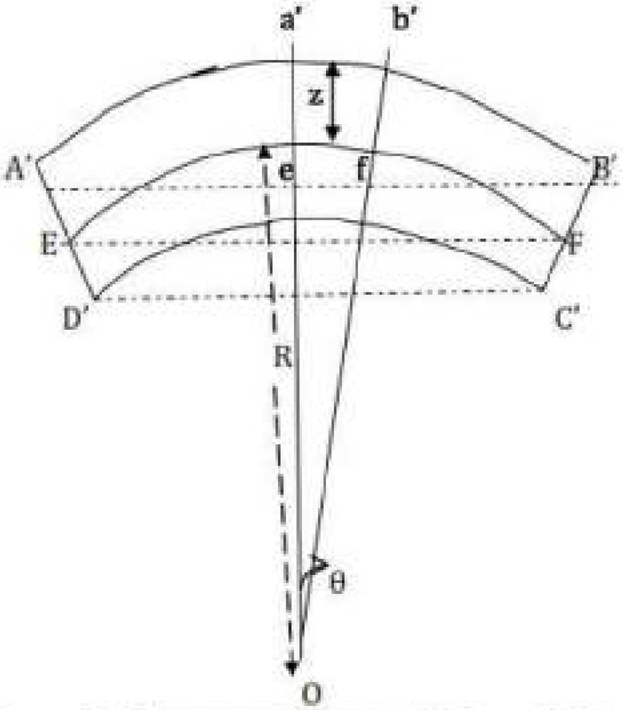
The beam can be imagined as made up of a number of longitudinal filaments. Under the applied load the upper filaments will undergo extensions and assume a convex form and the lower filaments will undergo compressions and assume a concave form. The filament that does not undergo any change in dimension is the neutral axis. The magnitude of extension or compression will depend on the distance of the filament from the neutral axis.

The moment of the restoring couple is called bending moment of a beam.

EXPRESSION FOR THE BENDING MOMENT OF A BEAM

Let ABCD be a section of the beam. EF is the neutral axis. Under the action of the external couple, the section will bend into an arc without any change in length of neutral axis.





The upper filaments will undergo extension and A'B' > AB.
The lower filaments will be subjected to compressive stresses and C'D' < CD

The beam is bent into an arc of a circle of radius 'R' with the center at O.

Consider a small portion of the neutral axis, ef , subtending an angle θ at the center. a'b' is another small portion of the filament a'b' which is at a distance z, from the neutral axis.

In the absence of bending, a'b' = ef = R θ and

a'b' = (R+z) θ

Strain in the filament, a'b' = {(R+z) θ – R θ} /Rθ = z/R,

If δA is the area of the filament a'b' , which is at a distance z from the neutral axis and Y is the Youngs modulus,

$$Y = \frac{\delta F/\delta A}{z/R}$$

Force acting on the filament, $\delta F = Y\, z\, \delta A\, R$ and Moment of the force about neutral axis = $Y z$. z The bending moment of the beam is obtained by summing over the moments of all the filaments above and below the neutral axis

Bending Moment $= \sum \frac{Y}{R}\, \delta A\, z^2 = \frac{Y}{R} \sum \delta A z^2 = Y I_g / R$

$$I_g = \sum \delta A\ z^2 = A\, k^2$$

is the geometrical moment of inertia and depends on the area of cross section, A of the beam and the radius of gyration k of the area about the neutral axis.

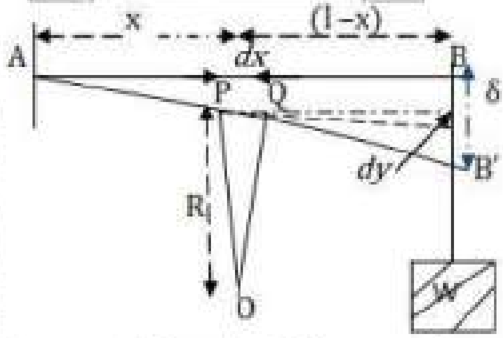
For rectangular cross section, $$I_g = b\,d^3/12$$

where, b is the breadth and d is the thickness of the beam.

For circular cross section, $$I_g = \pi\,r^4/4$$ where, r is the radius

EXPRESSION FOR YOUNG'S MODULUS OF A CANTILEVER:

A cantilever is a beam fixed at one end and loaded at the other end. Consider a load, 'W' applied to the free end of the cantilever of length, 'l'. Let δ be the deflection of the free end of the cantilever under the load. The deflection is maximum, at the free end and is equal to zero at the fixed end. Let PQ =dx ,be a small section of the neutral axis AB.



PQ is at a distance x from the fixed end.

Radius of curvature of neutral axis = R. Deflection of PQ = dy

The radius of curvature of the neutral axis is given by standard expression

$$\frac{1}{R} = \left[\frac{d^2y/dx^2}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}\right] = \frac{d^2y}{dx^2} \text{ ---------(i)}$$

dy/dx is very small and the higher power can be neglected.

The moment of applied force acting on the element PQ = W(l-x) ------(ii)

The restoring couple due to the elasticity of the beam =YI_g/R ---------(iii)

At equilibrium, these two are equal and opposite,

So,

$$YI_g/R = YI_g.\frac{d^2y}{dx^2} = W(l\text{-}x) \qquad \text{-------------(iv)}$$

$$YI_g \int \frac{d^2y}{dx^2}\, dx = w \int (l - x)dx$$

On integration, we obtain the expression

$YI_g \frac{dy}{dx} = w(lx - x^2/2) + C$ where C is the constant of integration.

The deflection is zero at x = 0 and dy/dx = 0 implies C=0.

Therefore

$$YI_g \int_0^{\delta} dy = w \int_0^{l} \left\{lx - \frac{x^2}{2}\right\} dx$$

$$YI_g\, \delta = w\left[\frac{lx^2}{2} - \frac{x^3}{6}\right]_0^l = w\left[\frac{l^3}{2} - \frac{l^2}{6}\right] = \frac{wl^3}{3}$$

$$\delta = \frac{wl^3}{3YI_g} \qquad \text{---------(v)}$$

For a rectangular cross section, $I_g = bd^3/12$, and

$$\delta = \frac{4wl^3}{Ybd^3}$$

Young's modulus is given by

$$Y = 4Mgl^3/bd^3\,\delta$$

MODULE-3

MAXWELL'S EQUATIONS – EM WAVES

INTRODUCTION:

Electromagnetics is the subject that deals with the theory and applications of electric and magnetic fields. The concept of electromagnetics is of prime importance in almost all fields of engineering, especially electromagnetic communication which forms the foundation of Mobile communication, Satellite Communication, Optical fiber communication etc.

In all these applications, Electric and Magnetic field properties are utilized to transfer the information.

Electricity Generator, Xerox machine, Printer, Cathode Ray oscilloscope, Ammeter, Voltmeter, Speakers, Hard disc of a computer, Electrocardiography (ECG), Transformers, Audio-Video recording, Magneto hydrodynamic (MHD) generator, Stealth aircraft are few more examples.

A course in Electromagnetism deals with electric and magnetic fields which are vectors. For the purpose of describing the properties of Electric and Magnetic fields, in 1881 J W Gibbs and Heaviside developed VECTOR ANALYSIS.

FUNDAMENTALS OF VECTOR CALCULUS:

Physical quantities can be divided into two main classes: (i) Scalars and (ii) Vectors.

Scalars: Those physical quantities which possess only magnitude and no direction in space such as temperature, mass, time, volume etc., are known as scalar quantities.

Vectors: Those physical quantities which have magnitude and direction both and also obey the parallelogram law of addition of vectors are called vector quantities; such as velocity, acceleration, force, field strength etc.

All the quantities which have magnitude and direction are not vectors.

For instance, electric current has magnitude and direction yet it is not a vector quantity as it does not obey the parallelogram law of addition.

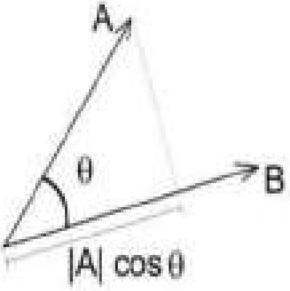
Importance of direction:

- Weather reporting requires direction of wind
- Pilot needs to know the direction of wind for proper flight plan
- Antenna detects electric field arriving in a specific direction
- Deflection of electrons in CRO depends on the direction of electric and magnetic fields
-

SCALAR PRODUCT OR DOT PRODUCT:

The scalar product of two vectors is defined as the product of the magnitude of the two vectors and the cosine of the smaller angle between them. It is represented by a dot between the vectors so it is also called as dot product. We can write

$$A \cdot B = A\,B\cos\theta$$

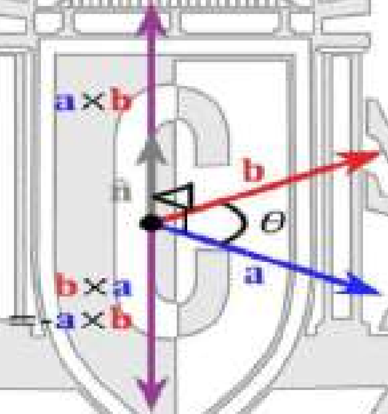


VECTOR PRODUCT OR CROSS PRODUCT:

The vector product of two vectors is defined as the vector having a magnitude equal to the product of the magnitude of the two vectors and the sine of the angle between them and is in the direction perpendicular to plane containing the two vectors.

$A X B = A B \sin\theta$

$\vec{A} X \vec{B} = |A|.|B| \sin\theta.\hat{n}$



OPERATION:

'Del' is a vector differential operator represented by the symbol ∇ (Nabla).
When applied to a function defined on a one-dimensional domain, it denotes its standard derivative as defined in calculus.
When applied to a field (a function defined on a multi-dimensional domain), del may denote the gradient (locally steepest slope) of a scalar field (or sometimes of a vector field, as in the Navier–Stokes equations), the divergence of a vector field, or the **curl** (rotation) of a vector field, depending on the way it is applied.

GRADIENT:

Gradient operation is performed to find the spatial variation of a scalar quantity such as temperature, height etc.

$$\nabla T = \frac{\partial T}{\partial x}\hat{a}_x + \frac{\partial T}{\partial y}\hat{a}_y + \frac{\partial T}{\partial z}\hat{a}_z$$

Gradient operation on a scalar yields a vector.

Ex: Electric field is expressed as gradient of potential

$$\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right)$$

Gradient vector always points in a direction showing maximum change of the scalar function.

$$dT = \nabla T \bullet dl = \left|\vec{\nabla} T\right|\left|\vec{dl}\right| \cos\theta$$

DIVERGENCE:

This operation is used to evaluate the amount of physical quantity emerging (diverging) from a small volume. From the definition of divergence, we construct:

$$\nabla \bullet \vec{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

Here, we observe that the divergence of a vector function $\vec{a}$ is itself a scalar.
Divergence is a measure of how much the vector spreads out from the point in question.

Ex- Positive charge is a source of divergence
Negative charge is a source of negative divergence
Tip of a fountain head is source of positive divergence

CURL:

In vector calculus, the **curl** is a vector operator that describes the infinitesimal rotation of a vector field in three-dimensional Euclidean space. At every point in the field, the curl of that point is represented by a vector. The attributes of this vector (length and direction) characterize the rotation at that point.

The direction of the curl is the axis of rotation, as determined by the right-hand rule, and the magnitude of the curl is the magnitude of rotation.

If the vector field represents the flow velocity of a moving fluid, then the curl is the **circulation density** of the fluid. A vector field whose curl is zero is called irrotational. The curl is a form of differentiation for vector fields.

The curl of a vector field **F**, denoted by curl **F**, or $\nabla \times \mathbf{F}$, or rot **F**, at a point is defined in terms of its projection onto various lines through the point. If $\hat{\mathbf{n}}$ is any unit vector, the projection of the curl of **F** onto $\hat{\mathbf{n}}$ is defined to be the limiting value of a closed line integral in a plane orthogonal to $\hat{\mathbf{n}}$ as the path used in the integral becomes infinitesimally close to the point, divided by the area enclosed.

$\nabla \times \mathbf{F}$ is, for **F** composed of $[F_x, F_y, F_z]$:

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

where **i**, **j**, and **k** are the unit vectors for the *x*-, *y*-, and *z*-axes, respectively.

Physical significance:

1. Curl measures the tendency for the vector field to swirl around.
2. Dot product checks "parallelness".
3. Cross product checks "perpendicularity"

LINE, SURFACE AND VOLUME INTEGRALS:

Let us now extend the concept of integration to cases in which the integrand involves a vector.

Line integral

By "line" we mean a path along a curve and space. Given a vector field A and curve L we define the integral.

$$\int A.dl = \int_a^b |A| \cos\theta \, dl$$

As the line integral of A around 'l'

Surface integral

Given a vector field A, continuous in a region containing the smooth surface S, we define the surface integral or the flux of A through S as

$$\psi = \int_S |A| \cos\theta \, dS = \int_S A.a_n \, dS$$

Volume integral

Closed surface defines a volume for which we define the integral

$$\int_V \rho V \, dV$$

As the volume integral of the scalar ρ_v over the volume V

LAWS OF ELECTROSTATICS:

Coulomb's law

The electrostatic force of Interaction between two point charges is
a. directly proportional to the product of their charges
b. inversely proportional to the square of the separating distance
c. acts along the position vector joining the two charges
d. is attractive for unlike charges and repulsive for like charges.

Force on charge q_2 due to q_1 separated by a distance r is

$$\vec{F} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{\left|\vec{R}_{BA}\right|^2}\hat{R}_{BA}$$

Does it work inside nucleus? at large distance ?

What if charges are in motion?

Electrical field Intensity:

- It is the force experienced by a unit positive charge.

$$\vec{E} = \underset{q\to 0}{Lt}\frac{F}{q}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}\left(\frac{1}{q_2}\right) = \frac{q_1}{4\pi\varepsilon|\vec{r}|^2}\hat{r}$$

- It can be represented as the gradient of potential along the direction of decreasing potential.

$$\vec{E} = -[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial v}{\partial z}]\hat{a}_z$$

Electric flux (Φ): Lines of force normal to the surface

Lines of force emanate from positive charge and converge on negative charge.

- **Faradays Law of electric flux** (not to be confused with the law of electromagnetic induction):

The number of lines of force emanating from a charge is directly proportional to the quantity of charge.

- **Electric flux density (D): [Electric displacement]**

- It represents the lines of force crossing unit area normally.

$$\vec{D} = \frac{flux(\Phi)}{area(A)}$$

If the flux lines are inclined to the normal, then

$$\vec{D} = \frac{flux(\Phi)}{area(A) \bullet \cos\theta}$$

$$\vec{D} = \varepsilon \vec{E}$$

Gauss Law:

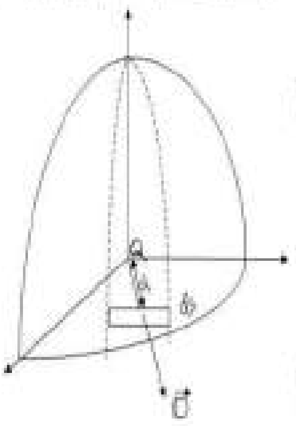
*(A method to find Electric Intensity which is **EASIER** than Coulomb's law but has limitations)*

The total electric flux over a closed surface is equal to the charge enclosed by the surface.

$$\bar{D} = \varepsilon \bar{E} = \frac{Q}{4\pi r^2}\hat{a}_n$$

$$\therefore \varphi = \iint \bar{D} \bullet d\bar{s} = \frac{Q}{4\pi r^2}\iint_s ds\hat{a}_n = \frac{Q}{4\pi r^2} \bullet 4\pi r^2$$

$$\left(\oiint_s D \bullet ds\right) = q$$



Total flux Φ = Q

$$\nabla \bullet \bar{D} = \rho$$

Applications: are only few….. Applicable only for symmetric charge distribution

Gauss Divergence Theorem:

Statement: "The volume integral of the divergence of a vector field A taken over any volume V bounded by a closed surface S is **equal to the surface integral of A over the surface S."**
Or

The volume integral of the divergence of a vector field over the volume enclosed by surface S is equal to the flux of that vector field **taken over that surface S."**

Hence, this theorem is used to convert volume integral into surface integral.

Mathematically,

$\iiint_V \text{div } A \, dv = \iiint_V (\Delta . A) \, d v = \iint_S A . ds$

Proof : Let a volume V) e enclosed a surface S of any arbitrary shape.

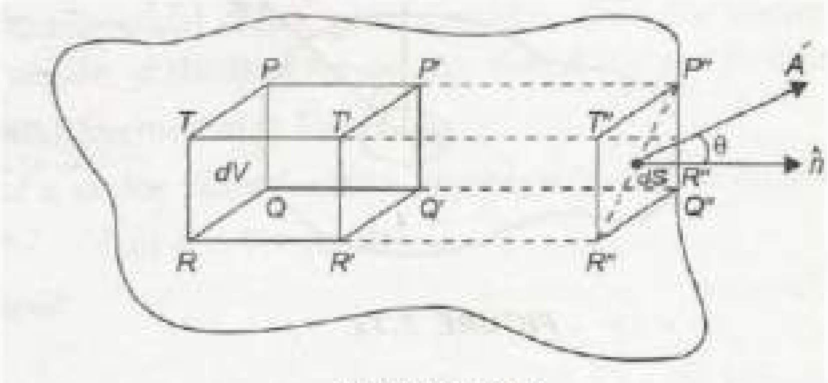


FIGURE 7.13

Let a small volume element PQRT T'P'Q'R' of volume dV lies within surface S as shown in Figure 7.13. As we know that flux diverging per unit volume per second is given by div Ai therefore, for volume element dV the flux diverging will be div AdV. If V be the volume enclosed by the surface S, then the total flux diverging through volume V will be equal to the volume integral.

$\iiint_V \text{div } A \, dV$

Now consider a surface element dS and n a unit vector normal to dS. Let θ be the angle between A and iz at dS, then A.dS will give the flux through the surface element dS. Where dS = n dS = area vector along n. Therefore, the total flux passing through the surface S may be obtained by the integral.

$\iint_S A . n \, d_S = \iint S \, A . dS$

But the total flux through the entire surface S [given by Equation (2)] must be equal to the total flux diverging from the volume V enclosed by surface S [given by Equation (1)] and therefore

$\iiint_V \text{div } A \, D_v = \iint S \, A . Ds$

Stokes theorem:

Curl of a vector over a surface is equal to the line integral of the vector along the line enclosing the surface.

$$\iint_s \nabla X H.ds = \int_l H.dl$$

Gauss flux theorem

We know that there is always a static electric field around a positive or negative electrical charge and in that static electric field there is a flow of energy tube or flux. Actually this flux is radiated /emanated from the electric charge. Now amount of this flow of flux depends upon the quantity of charge it is emanating from. To find out this relation, the **Gauss's theorem** was introduced. This theorem can be considered as one of the most powerful and most useful theorem in the field of electrical science. We can find out the amount of flux radiated through the surface area surrounding the charge from this theorem.

This theorem states that the total electric flux through any closed surface surrounding a charge is equal to the net positive charge enclosed by that surface. Suppose the charges Q_1, Q_2_ _ _ Q_i, _ _ _ Q_n are enclosed by a surface, then the theorem may be expressed mathematically by surface integral as

$$\psi = \int\int D.dS = \sum_{i=1}^{n} Qi$$

Where, D is the flux density in coulombs/m^2 and dS is the outwardly directed vector.

Gauss law of Magnetostatics:

In physics, **Gauss's law for magnetism** is one of the four Maxwell's equations that underlie classical electrodynamics. It states that the magnetic field **B** has divergence equal to zero.

$$\nabla \bullet B = 0$$

Ampere's law *(Magnetic counterpart of Gauss law):*

The integral around a closed path of the component of the magnetic field tangent to the direction of the path equals μ times the current intercepted by the area within the path

$$\oint B.dl = \mu I_p$$

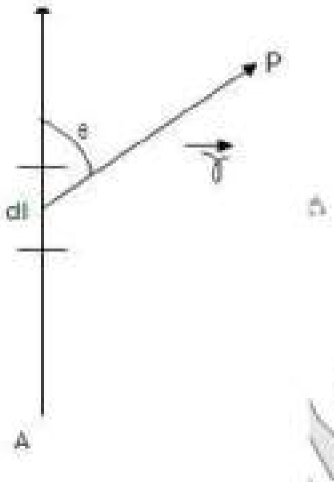
$$B = \mu H$$

$$\oint H.dl = I_p$$

Biot-Savart's Law

(Magnetic Counterpart of Coulomb's law)

The magnitude of the flux density at a point due to a current element is directly proportional to the product of the current (I), the length of the current **element (dl) and the sine of the angle (sinθ)** between the current element and the position vector to the point from the current element and it is inversely proportional to the square of the distance between the point and the current element.



$$\vec{dB} = \frac{\mu}{4\pi}\frac{I\,\vec{dl}\ X\ \hat{a}_r}{r^2}\quad T$$

$$\vec{dB} = \frac{\mu}{4\pi}\frac{I.dl.\sin\theta}{r^2}.\hat{n}$$

Faradays law of Electromagnetic Induction:

The magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux linking with it. The induced emf will be in a direction which opposes the change that causes it.

$$e = -\frac{d\phi}{dt}$$

$$e = \oint E \bullet dl = \iint \frac{dB}{dt}.ds$$

$$\iint \nabla X E.ds = -\iint \frac{\partial B}{\partial t}.ds$$

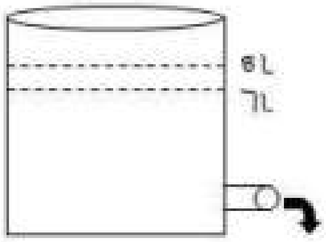
$$\nabla X E = -\frac{\partial B}{\partial t}$$

EQUATION OF CONTINUITY

(A fundamental equation in fluid dynamics)

The total amount (of the conserved quantity mass, energy, charge, momentum…) inside any region *can only change by the amount that passes in or out of the region through the boundary.* For ex:

1. Amount Water flowing out of a tank = Net decrease in quantity of water in the tank



2. For the case of flow of charge through a conductor

$$\nabla \bullet J = -\frac{\partial \rho}{\partial t}$$

Here J is current density and ρ is charge density

Consider a closed surface enclosing a charge Q with volume of charge density ρ. If there is any charge flow out of this surface, then charge flow through small area ds is $J \bullet ds$ where J is the current density

Total amount of charge flowing outwards = $\iint J \bullet ds$

Charge in the entire volume = $\iiint \rho\, dv$

As the charge flows outwards, the net charge present inside the volume decreases with time. Therefore, amount of charge flowing outwards = Decrease in the net charge inside the volume

$$\iiint \nabla \bullet J \;\; dv = -\frac{\partial}{\partial t}\iiint \rho\, dv$$

$$\nabla \bullet J = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t}(\nabla . D))$$

$$\nabla \bullet \left(J + \frac{\partial D}{\partial t} \right) = 0$$

Amperes law yields a result which is not in accordance with Equation of Continuity.

Ampere's law:

$$\nabla \times H = J$$

Take divergence on both sides

$$\nabla \bullet (\nabla \times H) = \nabla \bullet J$$

Since the divergence of a curl is zero

$$\nabla \bullet J = 0$$

Divergence of current density is zero

Equation of Continuity:

$$\nabla \bullet J = -\frac{\partial \rho}{\partial t}$$

Divergence of current density is equal to rate of decrease in charge density

DISPLACEMENT CURRENT

In free space, the displacement current is related to the time rate of change of electric field.

In a dielectric the above contribution to displacement current is present too, but a major contribution to the displacement current is related to the polarization of the individual molecules of the dielectric material. Even though charges cannot flow freely in a dielectric, the charges in molecules can move a little under the influence of an electric field. The positive and negative charges in molecules separate under the applied field, causing an increase in the state of polarization, expressed as the polarization density **P**. A changing state of polarization is equivalent to a current.

$$J = \frac{\partial D}{\partial t}$$

MAXWELL- AMPERE'S LAW:

Maxwell observed that Equation of Continuity is more sacred than Ampere's law.

Maxwell found that if a new term $\frac{\partial D}{\partial t}$ known as displacement current density is added to the right hand side of Ampere's law, it becomes adequate.

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Take divergence on both sides

$$\nabla \bullet (\nabla \times H) = \nabla \bullet \left(J + \frac{\partial D}{dt} \right)$$

Since the divergence of a curl is zero

$$\nabla \bullet \left(J + \frac{\partial D}{dt} \right) = 0$$

EQUATION OF CONTINUITY:

$$\nabla \bullet J = -\frac{\partial \rho}{\partial t}$$

From Gauss Law

$$\left(\nabla \bullet D \right) = \rho$$

$$\nabla \bullet J = -\frac{\partial}{\partial t}\left(\nabla \bullet D \right) = -\nabla \bullet \frac{\partial D}{dt}$$
$$\Rightarrow \nabla \bullet \left(J + \frac{\partial D}{dt} \right) = 0$$

Divergence of sum of current density and displacement current density is zero.

MAXWELL'S EQUATIONS:

These equations describe most of the natural phenomena. In Differential Form, Maxwell's equations are given as

$$\nabla \bullet \vec{D} = \rho$$

$$\nabla \bullet \vec{B} = 0$$

$$\nabla X \vec{H} = J + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla X \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

ELECTROMAGNETIC WAVES

Wave equation for electric field:

$$\nabla x E = -\frac{\partial B}{\partial t}$$

$$\nabla x(\nabla x E) = -\mu\frac{\partial}{\partial t}\left(\nabla x H\right)$$

$$\nabla x(\nabla x E) = \nabla\left(\nabla \bullet E\right) - \nabla^2 E = \nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2 E$$

$$\nabla \bullet E = \frac{\rho}{\varepsilon}$$

$$\nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2 E = -\mu\frac{\partial}{\partial t}\left(J + \varepsilon\frac{\partial E}{\partial t}\right)$$

$$\nabla^2 E - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} = \mu\frac{\partial J}{\partial t} + \nabla\left(\frac{\rho}{\varepsilon}\right)$$

$$\nabla^2 E - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} = 0$$

Plane waves:

These are the electromagnetic waves in which electric and magnetic fields are uniform over a plane perpendicular to the direction of propagation.

$$\nabla x H = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = -\frac{\partial H_z}{\partial x}\hat{y} + \frac{\partial H_y}{\partial x}\hat{z}$$

$$\nabla x H = \varepsilon\frac{\partial E_x}{\partial t}\hat{x} + \mu\frac{\partial E_y}{\partial t}\hat{y} + \mu\frac{\partial E_z}{\partial t}\hat{z}$$

$$-\frac{\partial H_z}{\partial x}\hat{y} + \frac{\partial H_y}{\partial x}\hat{z} = \varepsilon\frac{\partial E_y}{\partial t}\hat{y} + \varepsilon\frac{\partial H_z}{\partial t}\hat{z}$$

$$\frac{\partial H_z}{\partial x} = -\varepsilon\frac{\partial E_y}{\partial t}$$

$$\frac{\partial H_y}{\partial x} = \varepsilon\frac{\partial E_z}{\partial t}$$

$$\frac{\partial H_z}{\partial x} = -\varepsilon\frac{\partial E_y}{\partial x}\left(\frac{\partial x}{\partial t}\right) = -\varepsilon v\left(\frac{\partial E_y}{\partial x}\right)$$

$$H_z = -\varepsilon v E_y = -\sqrt{\frac{\varepsilon}{\mu}}E_y$$

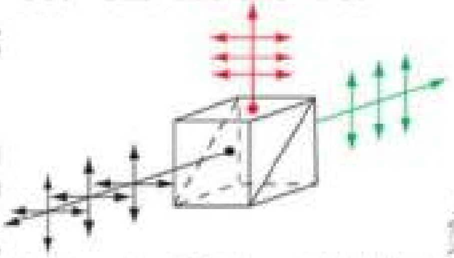
E AND H ARE PERPENDICULAR:

$$E \bullet H = \left(E_y\hat{y} + E_z\hat{z}\right)\left(H_y\hat{y} + H_z\hat{z}\right) = E_yH_y + E_zH_z = E_y\left(-\sqrt{\frac{\varepsilon}{\mu}}\right)E_z + E_z\left(\frac{\varepsilon}{\mu}\right)E_y = 0$$

TRANSVERSE NATURE OF EM WAVES:

In an electromagnetic wave, electric and magnetic field vectors are perpendicular to each other and at the same time are perpendicular to the direction of propagation of wave. This nature of electromagnetic wave is known as Transverse nature.

Maxwell proved that both the electric and magnetic fields are perpendicular to each other in the direction of wave propagation. He considered an electromagnetic wave propagating along positive x-axis. When a rectangular parallelepiped was placed parallel to the three co-ordinate axis, the electric and magnetic fields propagate sinusoidal with x-axis and are independent of y and z axis.
The figure shows rectangular parallelepiped along positive direction.



The rectangular parallelepiped does not enclose any charge thus the total electric flux across it must be zero. This law is called gauss's law, i.e

$\oint \vec{E}.\vec{ds}=0$

Have you ever anchored one end of a rope and held the other end in your hand? You can create a transverse wave by moving your hands in different direction.

The **electric field** remains the same at different points on x-axis; this proves that the electric field is static in nature. It is known that static fields can propagate as a wave, hence $E_X = 0$.

POLARIZATION OF PLANE WAVES:

The polarization state is one of the fundamental characteristics that is required to study light. There are three states of polarization that are used to describe light:

1. **Linear Polarization**
2. **Circular Polarization**
3. **Elliptical Polarization**

1. Linear polarization

If the polarization of all the electromagnetic waves in a light beam can be made so that each of the electric or magnetic field vector to have the same orientation, then the light beam is said to be polarized. Because of this, there is then a unique plane which contains all the directions of the electric or magnetic field along with the light rays. This type of polarization is referred to as plane polarization or linear polarization.

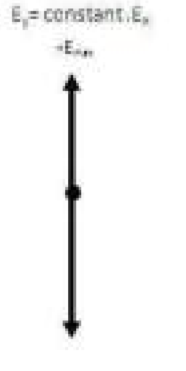
2. Circular Polarization

The second polarization state is referred to as circular polarization. Circular polarization can be described as the vectors of the electric field is rotated at a point in space in the direction that is perpendicular to the plane of propagation, instead of fixed orientation oscillation. The electric field vector magnitude also remains unchanged.

Circular polarization can be further classified according to the rotation. Looking at the oncoming light wave, if the electric field vector of the light appears to be rotated in a clockwise direction, then the wave is referred to as right-circularly polarized. On the other hand, if the light vector appears to rotate in a counterclockwise direction, then the wave is said to be left-circularly polarized.

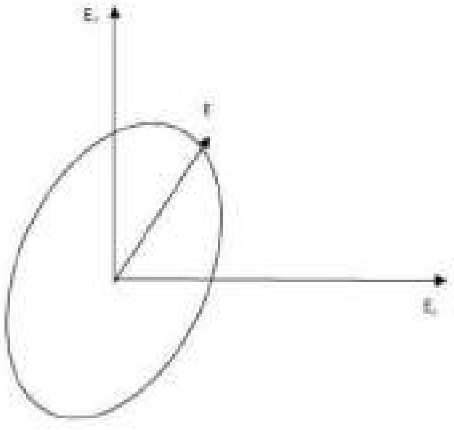
3. Elliptical Polarization

The third state of polrization is called elliptical polariztion. Elliptically polarized light consists of two light waves that are linearly polarized and having unequal amplitudes but has the same frequency. This results in a light wave with electric vectors that both rotates and changes its magnitude. An elliptical shape can be traced out by the tip of the electric field vector, and therefore it is referred to as elliptical polarization. It is also worthy to mention that circular polarization is a special case of elliptical polarization. The polarization of a uniform plane wave refers to the time varying behavior of the electric field at some point in space.



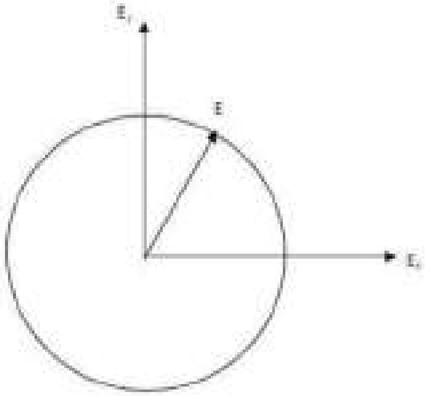
E_y= constant.E_x

Linear polarization



Phase difference between the X and Y components of electric field will be out of phase (not equal to 90^0)

Elliptical polarization



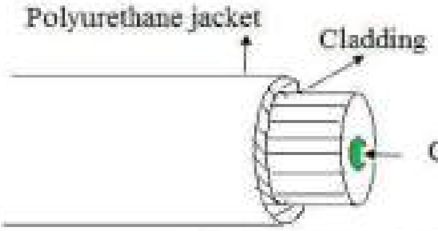
Phase difference between the components of electric field will be out of phase by 90^0

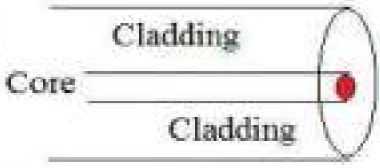
Circular polarization

OPTICAL FIBERS

- Fiber optics is an overlap of science and technology which deals with transmission of light waves into optical fibers, their emission and detection.
- It is a waveguide through which light can be transmitted with very little leakage through the sidewalls.
- These are essentially light guides used in optical communications as waveguides.
- ***The principle behind the transmission of light waves an optical fiber is TIR(Total internal Reflection)***
- They are transparent dielectrics and able to guide visible and infrared light over long distances.

CONSTRUCTION OF OPTICAL FIBER



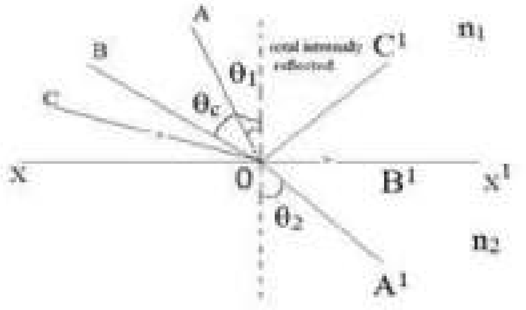


- An optical fiber is cylindrical in shape
- It has two parts **a)** inner part and **b)** outer part.
- The inner part is made of glass or plastic and its cylindrical in shape, it is called **core**. Core is having high refractive index.
- Outer part is a concentric cylinder surrounding the core, and is called **cladding**. Cladding is also made of same material with little lesser refractive index.
- The polyurethane jacket is used to enclose cladding which safeguards the fiber against chemical reaction with surroundings and also crushing.
- Many fibers which are protected by individual jackets are grouped to form a cable. A cable may consist of one to several hundred such fibers.

TIR (TOTAL INTERNAL REFLECTION)

It is the principle behind the transmission of light waves in an optical fiber which is a well-known optical phenomenon in physics. A ray **AO**, travelling in a medium of refractive index n_1 is separated by the boundary XX^1, from another medium of lower refractive index n_2. So $n_1 > n_2$.

The incident ray **AO** makes an angle θ_1 with the normal in the medium of refractive index n_1. The same **AO** undergoes **refraction** into the medium of refractive index n_2 and it bends away from the normal, since $n_1 > n_2$. θ_2 is the angle made by the refracted ray with the normal.



If we increase **θ_1** for certain value of **$\theta_1 = \theta_c$ called critical angle,** $\theta_2 = 90$, for such a case, the refracted ray grazes along the boundary of separation along **OB^1** while incident ray is along **BO.**

If **$\theta_1 > \theta_C$**, incident ray **CO** always gets reflected back into the same medium in which it is incident on the boundary. These takes place as per the law of reflection.

For refraction, we have the Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

For, $\theta_1 = \theta_C$ and $\theta_2 = 90^0$

$$n_1 \sin\theta_C = n_2 \sin 90^0 = n_2 \qquad (\sin 90^0 = 1)$$

$$\boxed{\theta_C = sin \frac{n_2}{n_1}}$$

PROPAGATION MECHANISM

Explain mechanism of light propagation in optical fibers.

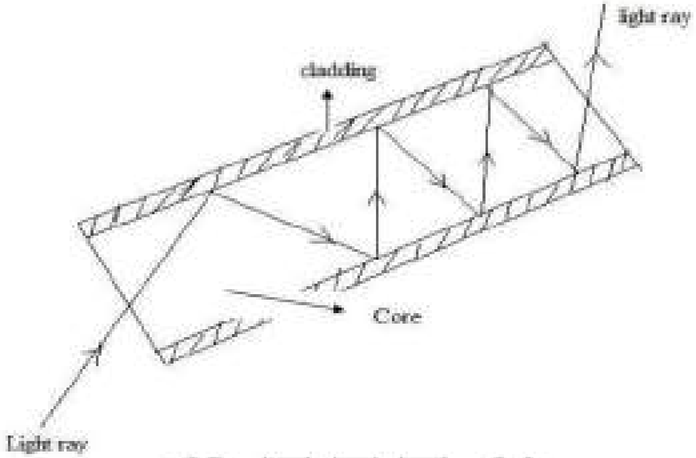
(Or)

Explain how optical fibers work as waveguides.

"Optical fibers are the devices used to transmit **light effectively along any desired path.**"

- Optical fibers work on the principle of total internal reflection (TIR)
- For total internal reflection there are two essential conditions, they are
  1) The light ray must pass from denser to rarer medium.

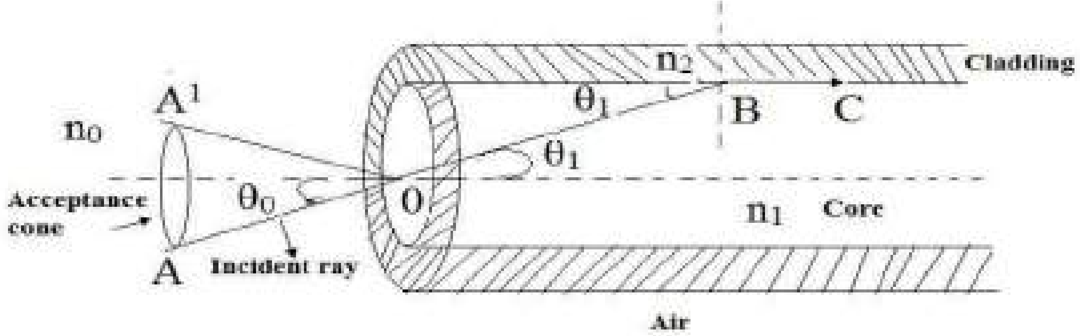
  2) The angle of incidence must be greater than the critical angle i > **c**



- A waveguide is a tubular structure through which energy of some sort could be guided in the form of waves.
- The waveguide as a light guide, also called fiber waveguide or fiber light guide
- An optical fiber consists of a core and cladding.
- In any optical fiber the refractive index of cladding is always lesser than that of its core to achieve TIR, i.e. R.I. (refractive index) of cladding < R.I. of core
- When a light is incident at one end of the fiber, it undergoes total internal reflection and finally emerges at the other end of the fiber. It is found that intensity of emergent light is almost same as that of incident light. In this way optical fibers transmit light effectively along any desired path.

RAY PROPAGATION IN THE FIBER, ANGLE OF ACCEPTANCE AND NUMERICAL APERTURE

- Consider a ray **AO** entering into the core at an angle θ_0 to the fiber axis. Then it is **refracted along OB** at an angle θ_1 in the core and further falls at critical angle of incidence (equal to $90 - \theta_1$) at **B** on the interface between core and cladding. Since the incidence is critical angle of incidence, the ray is refracted at 90^0 to the normal drawn to the interface i.e. grazes along BC.
- Any ray that enters into the core at an angle of incidence less than θ_0 will have refractive angle less than θ_1 because of which its angle of incidence $90^0 - \theta_1$ at the interface will become greater than the critical angle of incidence and hence undergoes total internal reflection.



- On the other hand any ray that enters at an angle of incidence greater than **θ₀**, will have to be incident at the interface at an angle less than the critical angle, it get **refracted i**nto the cladding region. Then it travels across the cladding and emerges into the surroundings and will be lost.
- If now **OA** is rotated around the fiber axis keeping **θ₀** same, it describes a conical surface.
- Therefore if a beam converges at a wide angle into the core, then those rays which are funneled into the fiber with in this cone will only be totally internally reflected, and thus confined within for propagation.

  θ₀ is called waveguide acceptance angle or the acceptance cone half angle.

  Sinθ₀ is called Numerical aperture (N.A) of the fiber.

The light gathering capacity of an optical fiber is known as Numerical aperture.

Condition for propagation:

Let **n₀, n₁, n₂** be the refractive indices of surrounding medium, core and cladding respectively.

For refraction at the point of entry of the ray **"AO"** into the core, we can apply **Snell's law, i.e.,** at point **A**

$$n_o \sin\theta_0 = n_1 \sin\theta_1 \quad \text{--------} (1)$$

At the point **B**

The angle of incidence =90^0-θ_1

Apply Snell's Law

$$n_1 \sin(90^0 - \theta_1) = n_2 \sin 90^0$$

$$n_1 \cos\theta_1 = n_2 \qquad (\sin 90^0 = 1)$$

$$\cos\theta_1 = \text{—} \quad \text{-------} (2)$$

Equation (1) can be written as

$$\sin\theta_0 = \frac{1}{\;} \sin\theta_1$$

$$= \frac{1}{}\sqrt{(\quad cos^2 \theta_1)}$$

$$= \frac{1}{n}\sqrt{1 \quad \frac{}{}\,)^2}$$

$$= \frac{1}{}\sqrt{\frac{}{}}$$

$$= \frac{}{} \frac{\sqrt{n}}{}$$

$$\boxed{\text{Sin } \theta_0 = \sqrt{\frac{}{}}}$$

If the medium surrounding the fiber is air then n_o=1

Therefore, Sin $\theta_0 = \sqrt{n \quad n}$

i.e.,

$$\boxed{\mathbf{N.A} = \sqrt{n \quad n}}$$

If $\boldsymbol{\theta_i}$ is the angle of incidence, then the ray will propagate if $\boldsymbol{\theta_i < \theta_0}$

(Or) $\sin\theta_i < \boldsymbol{\sin\theta_0}$

(Or) $\sin\theta_i < \sqrt{n \quad n}$

$$\boxed{\mathbf{\sin\theta_i < N.A}}$$

→Condition for propagation

Note: for light propagation, angle of incidence is less than $\boldsymbol{\theta_0}$

FRACTIONAL INDEX CHANGE (Δ):

The fractional index change **Δ** is the ratio of the refractive index difference between the core and cladding to the refractive index of core of an optical fiber.

$$\boxed{= \frac{n - n_2}{n_1} \text{ ---(3)}}$$

RELATION BETWEEN N.A. AND :

From equation (3) $(n_1 - n_2) = n_1 \Delta$ -------- (4)

$$\text{N.A} = \sqrt{n_1^2} - n$$

$$= \sqrt{(n_1 + n_2)\,(n_1 - n_2)}$$

$$= \sqrt{(n_1 + n_2)\,n} \qquad \text{from eq(4)}$$

Since, $n_1 \approx n_2$, $(n_1 + n_2) = 2n_1$

i.e N.A $= \sqrt{2n}$

$$\boxed{\text{N.A} = n_1 \;\overline{\quad}}$$

MODES OF PROPAGATION:

Mode can be defined as:

- The pattern of motion in a vibrating body.
- The light ray paths along which the waves are in phase inside the fiber.
- In simple terms these modes can be visualized as the possible number of allowed paths of light in an optical fiber.

Through it is expected that all the rays which enter into the core at an angle less than the acceptance should travel in the core, it is not even theoretical. By the application of Maxwell's equation, we can get to know that, out of the light that enters into the core within the waveguide acceptance angle, only the light waves in terms of certain number of modes will be sustained for propagation in the fiber.

V-NUMBER:

"The number of modes supported for the light propagation in the optical fiber is known as V-number."

V – Number is given by

$$V = \frac{\pi d}{} \sqrt{n_1^2} - n$$

Where, d is the diameter of the core, **λ** is the wavelength of light

n_1 is the R.I of the core n_2 is the R.I of the cladding

(Or) $V=\frac{\pi d}{\ }\ N.A$

If the fiber is surrounded by a medium of R.I n_0, then the expression is

$$V=\frac{\pi d}{\ }\sqrt{\frac{n\quad -n_2}{n_0}}$$

For V>>1

Number of modes = —

TYPES OF OPTICAL FIBERS:

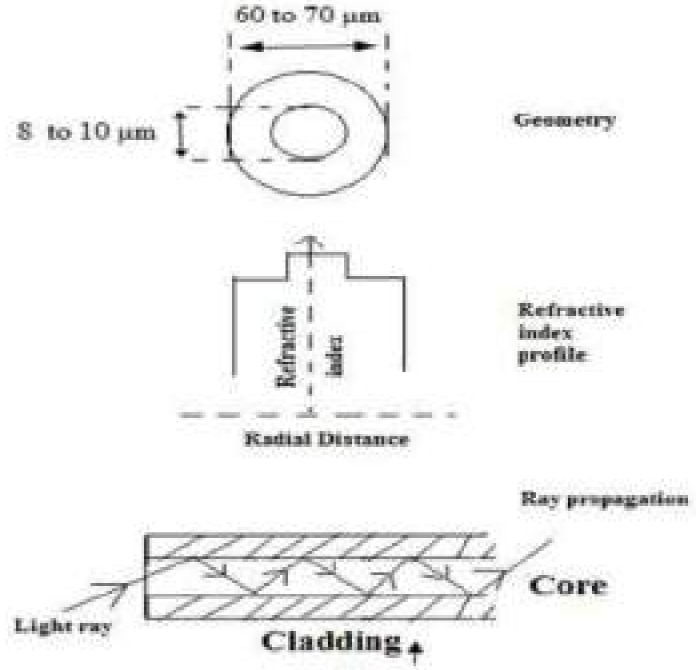
- Optical fibers are classified into 3 major categories based on the materials used for making optical fibers, number of modes transmitted and the R.I. profile of the fibers.
- In any optical fiber, the whole material of the cladding has a uniform refractive index value but the refractive index of core material may either remain constant or subjected to variation in a particular way. (R.I. of the core changes in graded index multimode fiber)
- The curve which represents the variation of refractive index with respect to the radial distance from the axis of the fiber is called the refractive index profile.

Optical fibers are classified into 3 categories namely:

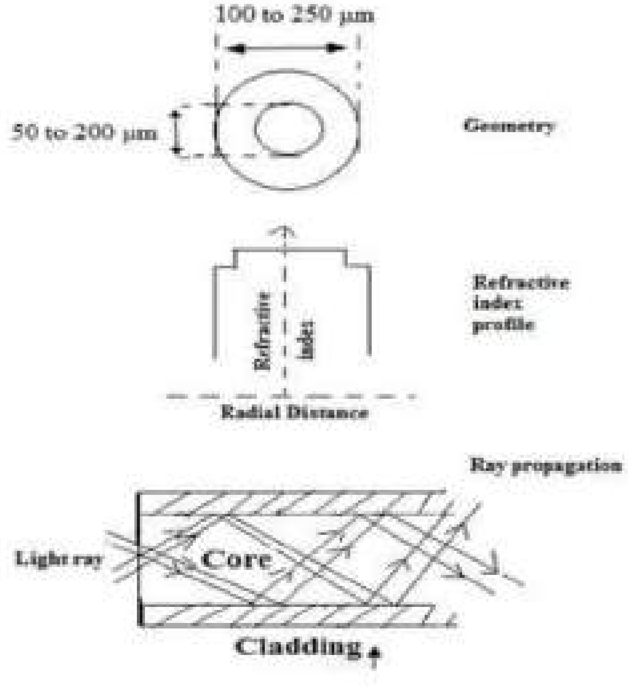
a) Single mode fiber
b) Step index multimode fiber
c) Graded index multimode fiber

Single mode fiber:

- Here core material has uniform refractive index value.
- Cladding also has uniform refractive index but of little lesser value than that of core. This results in a sudden increase in the value of R.I from cladding to core.
- R.I. profile takes the shape of a step.
- Diameter of the core is 8 to 10 μm and diameter of the cladding is around 60- 70 μm. Since the core is very narrow, it can guide just a single mode. Hence it is called single mode fiber.
- These are the most extensively used ones and constituent 80% of all the fibers that are manufactured.
- They need lasers as the source of light.
- It is less expensive, but very difficult to splice.
- Used in submarine cable system.

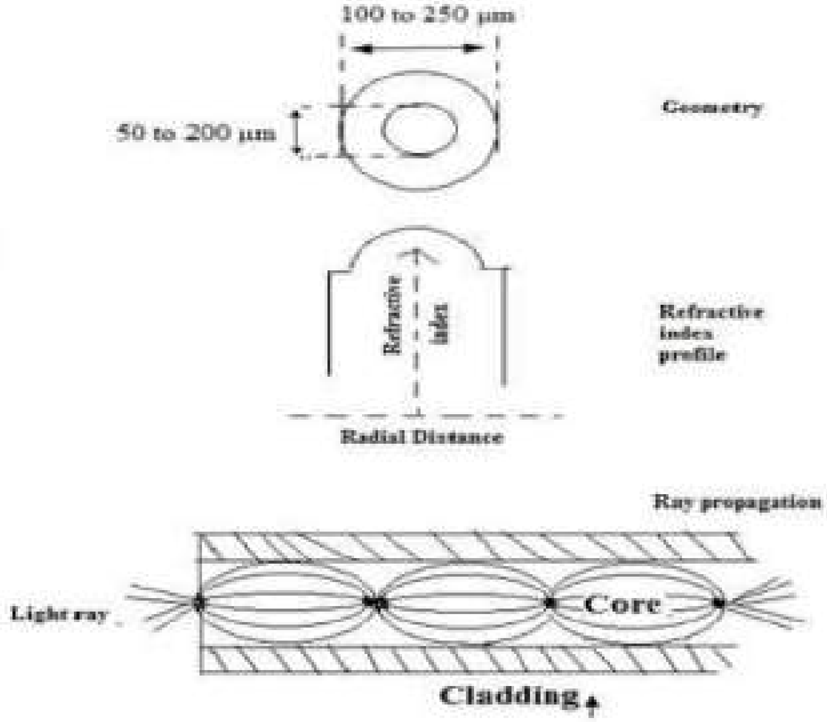


Step index multimode fiber:



- Here, the core material has uniform refractive index value.
- Cladding also has uniform refractive index but of little lesser value than that of the core. This results in a sudden increase in the value of R.I. from cladding to core.
- R.I. profile takes the shape of a step.
- Diameter of the core is 50 to 200 μm and diameter of cladding is 100- 250 μm.
- Here the core material has a much larger diameter, which supports propagation of large number of modes.
- R.I. profile is also similar to single mode optical fiber.
- Uses LED or laser as source of light.
- It is least expensive all and is used in data links which has lower band width requirements

Graded index multimode optical fiber:



- It is also denoted as GRIN.
- The geometry of GRIN is same as that of step index multimode fiber.
- The special feature of the core is that; its R.I. value decreases in the radially outward direction from the axis, and becomes equal to that of the cladding at the interface. But the R.I. of the cladding remains uniform.
- Diameter of the core 50 to 200 μm and diameter of cladding 100- 250 μm.
- Uses LED or laser as source of light
- Application is in the telephone trunk between central offices.

ATTENUATION (POWER LOSS OR FIBER LOSS):

The power loss suffered by the signal when it propagates through the fiber is called **Attenuation**. It is also known as fiber loss.

Types of losses in fiber are:

i) Absorption

ii) Scattering

iii) Radiation loss

i) Absorption loss:

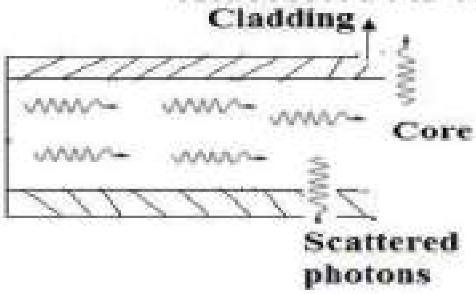
a. Absorption by impurities: Iron, Chromium, Cobalt and Copper are some of the impurities generally present in the glass fiber. When signal propagates through the fiber, a few photons associated with the signal are absorbed by the impurities present in the fiber. This results in power loss.

b. Intrinsic absorption: The absorption that takes place in the fiber material assuming that there are no impurities in it.

ii) Scattering loss:

a. Rayleigh scattering:

When a signal propagates through the fiber, a few photons associated with the signal are scattered by the scattering objects such as impurities present in the fiber. The dimensions of the scattering objects are very small compared to the wavelength of light. This type of scattering is similar to Rayleigh scattering. It is found that the co-efficient of scattering is inversely proportional to the wavelength of the object.

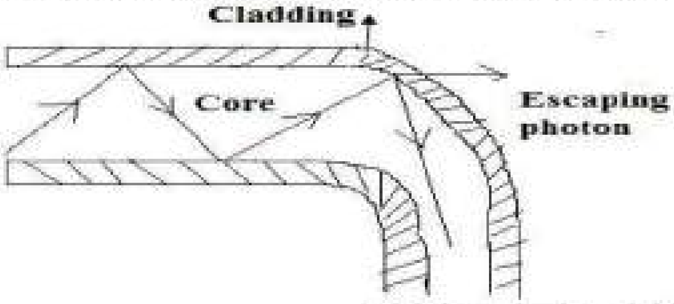


b. Others: Scattering also occurs due to trapped gas bubbles, unreacted starting materials and some crystallized region in the glass.

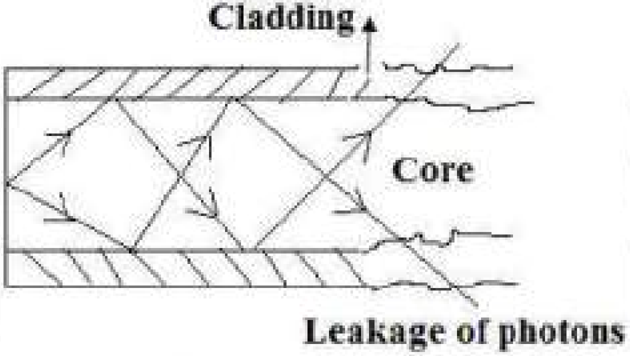
iii) Radiation loss:

It is due to the bending of fibers and it can be explained as follows:

a) **Macroscopic bending**: They are the bends with radii much larger compared to fiber diameter. It occurs while wrapping the fiber on a spool or turning it around a corner. If the bending is too sharp then the power loss becomes very high.



b) **Microscopic bending**: It occurs due to the non-uniformity in the fibers while manufacturing. Because of this a few modes undergo leakage which results in power loss.



<u>EXPRESSION FOR ATTENUATION CO-EFFICIENT (α)</u>:

By lamberts law "The rate of decrease of intensity with distance (—) is directly proportional to the initial intensity P".

i.e. $- — \alpha$ P ------ (1)

(-ve sign indicates that it is a decrement)

$- —$ P

Where α is a constant called Attenuation co-efficient.

$— = - \alpha\, dL$

By integrating both sides we have

$\int — = - \alpha \int dL$ -------- (2)

An optical fiber of length L

If P_{in} is the initial intensity with which the light is launched into the fiber and P_{out} is the intensity of the light received as output end of the fiber.

Equation (2) $\int_{pin} \; — = - \alpha \int_0 dL$

$= - \alpha L$ (Apply the lower limit and upper limit)

$\log_e p_{out} - \log_e P_{in} = - \alpha L$

$\alpha = - - \log_e \left[\frac{out}{}\right]$

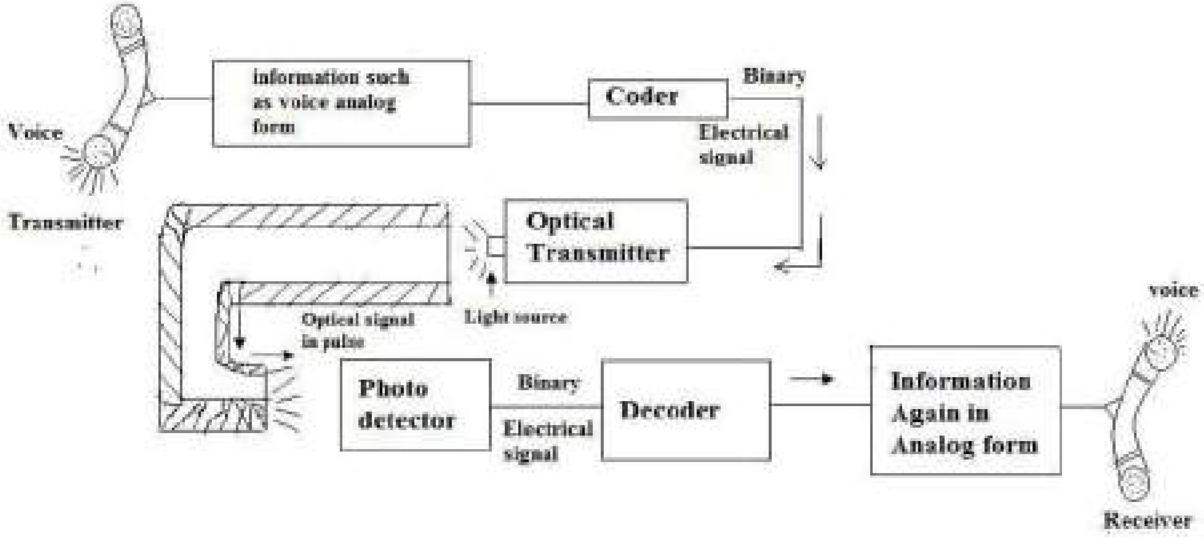
$\alpha = - - \log_{10} \left[\frac{out}{}\right]$ --- Bel /unit length

An optical fiber technology it is expressed α in terms of decibel / Kilometer (dB/Km). hence it follows that

1) The length of the fiber is expressed in Km.
2) The unit of Bel is 10 dB (1 Bel=10 Decibel)

$\alpha = - \frac{1}{} \log_{10} \left[\frac{P_{out}}{P_{in}}\right] \text{X } 10\text{dB/Km}$

DISCUSS THE POINT TO POINT COMMUNICATION SYSTEM USING OPTICAL FIBERS:



Optical fiber communication is the transmission of information by propagation of optical signal through optical fibers over the required distance which involves driving optical signal from electrical signal at the transmitting end and conversion of optical signal back to electrical signal at the receiving end.

- Firstly we have analog information such as voice of a telephone user. The voice gives rise to electrical signals in analog form coming out of the transmitter section of the telephone.
- The analog signal is converted to binary data (digital) with the help of an electronic system called coder.
- These electrical pulses are converted into optical pulses by modulating the light emitted by an optical source, in the binary form. This unit is called optical transmitter(converts electrical signals into light signals)
- This optical Signal is fed into the fiber.
- Out of the incident light which is funneled into the core within the half angle acceptance cone, only certain modes will be sustained for propagation within the fiber by means of total internal reflection. While propagating signal undergoes attenuation and delay distortion. Delay distortion is the reduction in the quality of signal with time. These effects cause degradation of the signal as the light propagates and may reach a limiting stage beyond which it may not be possible to retrieve the information from the light signal.
- The receiver section uses photo detector which converts the optical signal into corresponding electrical signal then electrical signal is amplified and recast in the original form by means of an electrical regenerator, which is part of receivers section.
- Lastly using the decoder, the binary electrical signal is converted back to analog electrical signal, which will be same information such as voice, which was there at the transmitting end.

MERITS OF OPTICAL FIBERS

1. The materials used for optical fibers are silicon dioxide and plastic, both of which are easily available at low cost.
2. Because of their compactness and light weight, fibers are must easier to transport.
3. The signals generated from radio and telecommunication stations or from some electronic equipments or sometimes radiation from lightening or sparking tend to cause disturbance in the metallic cable, but cannot do so for the fiber cable.
4. Since the signal is optical no sparks are generated as it could be in the case of electrical signal, hence it leads to protection from corrosive and flammable environments.

DEMERITS OF OPTICAL FIBERS

1. Fibers cannot be bent too sharply, for sharper bends either the fiber gets broken or light fails to undergo Total Internal Reflection.
2. The optic connectors used to connect two fibers (splicing) are highly expensive.

3. Whenever a fiber suffers a line break the operations required to reestablish the connections are highly skillful and time consuming.
4. Fibers undergo expansion and contraction with temperature that upset some critical alignments which leads to loss in signal power.

MODULE-4

QUANTUM MECHANICS

Quantum Mechanics is a new branch of study in physics which is indispensable in understanding the mechanics of particles or bodies in the atomic and the subatomic scale. The term quantum mechanics was first introduced by Max Born in1924.Within the field of engineering, quantum mechanics plays an important role. The study of quantum mechanics has led to many new inventions that include the laser, the diode, the transistor, the electron microscope, and magnetic resonance imaging. Flash memory chips found in USB drives also use quantum ideas to erase their memory cells. The entire science of Nanotechnology is based on the quantum mechanics.

WAVE NATURE OF PARTICLES

Protons, neutrons, electrons and photons are known as material particles.

According to de Broglie hypothesis, material particles in motion are associated with a wave known as **matter waves or de Broglie waves.**

When a particle has momentum 'p', the **De Broglie wavelength** of the matter wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

The above equation is known as de Broglie wave equation.

Hence de Broglie wavelength depends upon the mass of the particle and its velocity.

HEISENBERG'S UNCERTAINTY PRINCIPLE

One of the fundamental principles of quantum mechanics is the "Heisenberg's uncertainty principle". The principle was formulated in 1927 by German physicist Werner Heisenberg. It is also called the indeterminacy principle.

Statement:

"It is impossible to determine precisely and simultaneously both the position and momentum of a particle."

Further, "in any simultaneous determination of the position and momentum of a particle, the product of the corresponding uncertainties in the measurement is equal to, or greater than (h/4π)".

$$\Delta x \, \Delta p \geq h/_{4\pi}$$

Where **Δx** represents the uncertainty in the measurement of the position of the particle and **Δp** represents the uncertainty in the measurement of its momentum.

Heisenberg's uncertainty principle could also be expressed in terms of the uncertainties involved in the measurement of the physical variable-pair, **energy (E) and time (t) and also angular displacement (θ) and angular momentum (L).**

If ΔE and Δt are the uncertainties involved in determining the energy and time respectively, then

$$\Delta E\ \Delta t \geq h/4\pi$$

Similarly, if Δθ and ΔL are the uncertainties involved in determining the angular displacement and angular momentum of the particle respectively, then

$$\Delta\theta\ \Delta L \geq h/4\pi$$

where, the notation **Δ** associated with the respective variables indicates the minimum uncertainty involved in the measurement of the corresponding variable.

PHYSICAL SIGNIFICANCE OF HEISENBERG'S PRINCIPLE:

- It signifies that one should not think about accurate values for the position and momentum of a particle. Instead one should think about only the probable values for the position and momentum.
- The estimation of such probabilities are made by mathematical functions named probability density functions.

APPLICATION OF HEISENBERG'S UNCERTAINTY PRINCIPLE:

Non-existence of electrons in the nucleus:

As an application of Heisenberg's uncertainty principle, we consider the case of "Non-existence of electrons in the nucleus". This is based on the observations regarding the emission of β–rays during nuclear decay.

. Becquerel demonstrated that β-rays are actually streams of **electrons**. During β decay process it was found that the emitted β-rays travelled with different velocities. Becquerel demonstrated that β-rays are actually streams of **electrons** Their kinetic energy varied from very low values to a maximum value of about 3-4 MeV. Hence the obvious question, ***"is it possible that electrons exist inside the nucleus with certain energy, and the same energy appears as their kinetic energy when they are emitted?"***

According to Einstein's relativistic energy-momentum equation

$$E^2 = c^2\ [p^2 + m_o^2 c^4]\ ....(1)$$

where
m_0 = rest mass of electron = 9.1×10^{-31} kg,
p = momentum and
c = speed of light = 3×10^{8} m/s.

Now, for an electron to exist inside the nucleus, the uncertainty Δx in its position cannot be greater than the nuclear radius. The nuclear radius is in the order of 5×10^{-15} m. We can take this value to be approximately the maximum space of confinement. Thus the maximum value for uncertainty in position can be taken as

$$(\Delta x)_{\max} \leq 5 \times 10^{-15} \text{ m}$$

By uncertainty principle,

Δx —

$$_{\min} \geq \frac{h}{4\pi} \times \frac{1}{\Delta x}$$

$$\frac{6.63\times10^{-34}}{4\times3.14\times5\times10^{-15}}$$

$(\Delta p)_{\min}$ 1.1×10^{-20} Ns ....(2)

This is the uncertainty in the momentum of the electron. But since the momentum of the electron must atleast be equal to the uncertainty in the momentum, we can take

$(p_x)_{\min} \geq 1.1 \times 10^{-20}$ Ns ....(3)

We know that the rest mass of the electron is $m_o = 9.11 \times 10^{-31}$ kg.

Using the inequality (3) in equation (1), we can say that for the electrons to exist inside the nucleus, its energy E must be such that

$$E^2 \geq c^2 [p^2 + m_o^2 c^4]$$

$$(E)_{\min} \quad (3 \times 10^8)^2 [(1.1 \times 10^{-20})^2 + (9.11 \times 10^{-31})^2 (3 \times 10^8)^4]$$

Since the second term inside the bracket is too small, it is neglected.

$$\mathbf{(E)_{\min} \geq 20.6 \text{ MeV}}$$

This means to say that, for electrons to exist inside the nucleus, it must have a minimum energy of about 20.6 MeV. But beta-**decay studies indicate that the kinetic energy of β**-particles is of the order of 3 to 4 MeV. Hence, we can conclude that electrons cannot exist inside the nucleus.

WAVE FUNCTION:

In quantum mechanics, because of the wave-particle duality, the properties of particles can be described by a wave. Therefore its quantum state can be represented by a wave of any arbitrary shape. This is called a **Wave function.**

The variable quantity characterizing matter waves is known as **wave function.** It gives complete information about the system .It is also called state function as it speaks about the physical state of the system.

Wave function is obtained by solving a fundamental equation called **Schrodinger equation.**

- It varies w.r.t position and time.
- It is denoted by a symbol (Ψ)
- Ψ itself cannot be an observable quantity hence, Ψ has no direct physical significance
- Ψ can have real values or imaginary(complex).

<u>Properties of wave function</u>

Property 1: ψ is single valued everywhere.

A function f(x) which is not single valued over a certain interval is shown in Fig. 1. Here f(x) has three values, f_1, f_2 and f_3 for the same value of P at x = P. Since $f1 \neq f2 \neq f3$, it means to say that, if f(x) were to be wave function, then the probability of finding the particle has three different values at the same location P which is absurd. Hence this wave function is not acceptable.

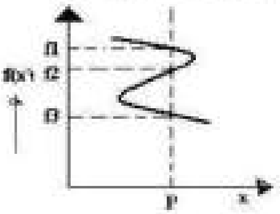


Figure: 1

Property 2: ψ is finite everywhere.

A function f(x) which is not finite at x = R is shown in Fig.2. At x = R, f(x) = ∞. Thus, if f(x) were to be a wave function, it signifies a large probability of finding the particle at a single location (x = R), which violates the uncertainty principle. Hence the wave function becomes unacceptable.

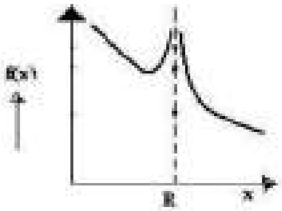


Figure: 2

Property 3: ψ and its first derivatives with respect to its variable are continuous everywhere.

A function f(x) which is discontinuous at Q is shown in Fig. 3. At x = Q, f(x) is truncated at A, and restarts at B. The function is not defined between A & B. If f(x) is taken to be a wave function, then the state of the system at x = Q cannot be ascertained. Hence the wave function is not acceptable.

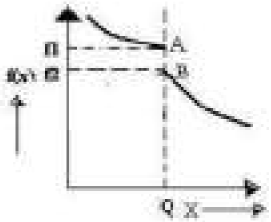


Figure 3

Also, when ψ is not continuous, its first derivatives will not be finite. The first derivatives should also be continuous, b**ecause, if they are not continuous, the second derivatives of ψ will not be** finite.

Property 4: For bound states, ψ must vanish at infinity. If ψ is a complex function, then ψ* ψ must vanish at infinity.

The wave functions that possess all the above properties are called **Eigen functions**.

PHYSICAL SIGNIFICANCE OF WAVE FUNCTION (ψ)

a) It gives the idea about the probability of finding a particle.

b) Let us consider a system of electrons. If ψ is the wave function associated with that system, then $|\psi|^2$ gives measure of density of electrons.

c) If we consider an electron to be present in the region of volume V then $|\psi|^2 dV$ gives the probability of finding the electron in the region of volume dV

d) The certainty of finding the electron in the entire region of volume V is given by

$$\int |\psi|^2 dV = 1$$

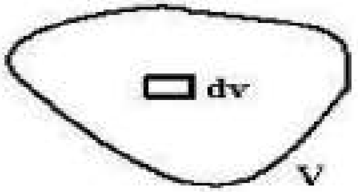
This is known as normalization.

Probability density:

According to Max born approximation, probability of locating a particle at a point is directly propotional to $|\psi|^2$. This is called **probability density.**

If $|\psi|^2 = 1$; then we are sure that particle is present .

If $|\psi|^2 = 0$; then the particle is completely absent.



Consider a particle inside the volume V. Let dV be the infinitesimally small element in V. If ψ is the wave function associated with the particle, then $|\psi|^2$ is the probability per unit volume of **space, centered at a point where ψ is evaluated at that time**

Thus, the probability of finding a particle is

$$P = \int |\psi|^2 dV$$

Here, the product of ψ. ψ^* is real and is written as $|\psi|^2$, **where** ψ^* is the complex conjugate of **ψ.**

Probability density is given by

$$|\psi|^2 = \psi\,\psi^*,\quad \text{where } \psi^* \text{ is the complex conjugate of } \psi$$

Normalization condition:

Further, if we are certain that the particle is present in a particular region or space of volume V, then

$$\int |\psi|$$

This value 1 for probability means, it is clearly a certainty. However, if we are not certain about locating the particle anywhere in the given volume, then it is expected to be present somewhere in space. Then the probability of finding the particle somewhere in the universe must be unity. Thus

$$\int |\psi|$$

This condition is known as the normalization condition. A wave function that satisfies the above condition is said to be normalized.

Eigen function and eigen value

Eigen functions are those functions which possess the properties of that they are single valued & finite everywhere and also their first derivatives with respect to their variables are continuous everywhere

To find ψ Schrodinger's wave equation has to be solved. Since it is a second order differential equation, hence it has many numbers of solutions. But all of them may not be the correct solutions in such a case we have to choose only a permitted function. These permitted functions are called Eigen function the corresponding energy values are called **Eigen values**

Operator.

An operator(Â) is a symbol or a code which directs one to perform an operation on the Eigen function (ψ)to get the information (Eigen value)(λ)

$$\hat{A}\psi = \lambda\psi$$

TIME INDEPENDENT SCHRODINGER WAVE EQUATION:

Schrodinger equation is a fundamental wave equation in quantum mechanics capable of determining the wave function ψ for different physical conditions.

According to de- Broglie theory, for a particle of mass 'm' moving with a velocity 'v', the wave length λ is given by,

– — ------------------(1)

where , p is the momentum of the particle.

The de- Broglie wave equation for a particle travelling in positive x-direction can be written in complex form as

$$\Psi = A\,e^{i(kx-\omega t)} \text{ ------------------ (2)}$$

where, A is the amplitude of the wave, ω is angular frequency of the wave, ψ is the total wave function and k is the wave number.

The space independent part in equation (2) can be represented as a wave function,

$$\Psi = \psi A\,e^{-i\omega t} \text{ ------------------ (3)}$$

Differentiate (3) twice wrt x. Since Ψ depends on x, we get

$$\frac{d^2\Psi}{dx^2} = e^{-i\omega t}\,\frac{d^2\psi}{dx^2} \text{ ---------- (4)}$$

Now again differentiate (3) w.r.t. time 't' twice, we get

$$\frac{d\Psi}{dt} = (-i\omega)\,Ae^{i(kx-\omega t)}\,\psi$$

$$\frac{d^2\Psi}{dt^2} = (-i\omega)\,(-i\omega)\,Ae^{i(kx-\omega t)}\,\psi$$

$$\frac{d^2\Psi}{dt^2} = -\omega^2 e^{-i\omega t}\ \psi \quad \text{-------------- (5)}$$

The equation for a travelling wave in differential form can be written as,

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

Then by analogy, the wave equation for a de Broglie wave for the motion of a free particle can be written as

$$\frac{d^2\Psi}{dx^2} = \frac{1}{v^2}\frac{d^2\Psi}{dt^2} \quad \text{------------------ (6)}$$

This equation represents the waves propagating along x-axis with a velocity 'v', and 'x' is the displacement at time 't'.

Substituting equations (4) and (5) in the above equation, we get

$$e^{-i\omega t}\,\frac{d^2\Psi}{dx^2} = \frac{1}{v^2}\,(-\omega^2 e^{-\omega t}\psi)$$

$$\frac{d^2\Psi}{dx^2} = \frac{-\omega^2}{v^2}\Psi$$

But, we know that ω = 2πν and v = νλ. Substituting for ω and v, we get

$= \psi$------- (7)

But, we have, . Then eq.(7) becomes

--------------(8)

We know that,

Total Energy = Kinetic Energy + Potential Energy

E = KE +PE

E =

-----------(9)

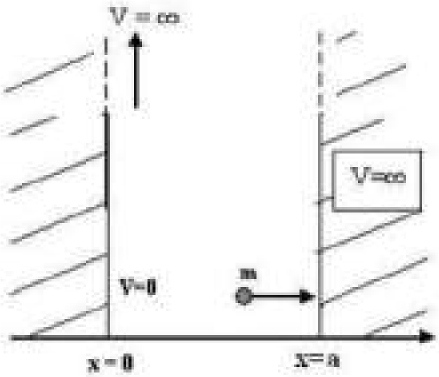
Substitute eq. (9) in (8), then

This is known as time independent Schrodinger wave equation in one dimension.

APPLICATIONS OF SCHRODINGER'S EQUATION

PARTICLE IN ONE DIMENSIONAL POTENTIAL WELL OF INFINITE HEIGHT

Consider a particle of mass 'm' moving inside a box along the X – direction between two rigid walls A and B. The particle is free to move between the walls of the box at x = o and x = a. The potential energy of the particle is considered to be zero inside the box and infinity at all points outside the box. This means that



1. PE, V=0 for $0 < x < a$
2. PE,V=∞ for $x \leq 0$ and $x \geq a$

The particle is always inside the box and therefore the probability of finding the particle outside the box is zero. Therefore the wave function ψ is zero for $x \leq 0$ and $x \geq a$.

The Schrodinger's equation for a particle is given by

Since, V = 0 inside the box between the walls, the above equation reduces to

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 mE}{h^2}\psi = 0 \text{ ---------- } \mathbf{(1)}$$

Take, **---------------- (1A)**

Then, equation (1) becomes

The general solution for equation (2) is of the form

$$\boldsymbol{\psi = A \sin kx + B \cos kx} \text{ --------------- (3)}$$

where A and B are constants. The values of these constants can be evaluated by applying the following boundary conditions.

The particle cannot penetrate the walls. Hence

(i) $\psi = 0$ at $x = 0$

$\therefore (3) \Rightarrow 0 = A \sin 0 + B \cos 0$

$\Rightarrow 0 = 0 + B \Rightarrow \boxed{\mathbf{B = 0}}$

$\therefore \psi = \mathbf{A\ Sinkx}$ ------------- (4)

Again, (ii) $\psi = 0$ at $x = a$

$(3) \Rightarrow 0 = A \text{ Sin } ka$

But A ≠ 0 ; because if A = 0, then the entire function will become zero.

Therefore, Sin ka = 0 = Sin nπ

$ka = n\pi$, $n = 0,1,2,3,\ldots\ldots$

$\Rightarrow$ — ------------------ (5)

Equation (4) becomes

$\therefore \psi_n = A \sin\left(\frac{n\pi}{a}\right)x$ ------------------ (6)

Substituting equation (5) in (1A), we get

$$\Rightarrow \frac{n^2\pi^2}{a^2} = \frac{8\pi^2 m E_n}{h^2}$$

$$\Rightarrow E_n = \frac{n^2h^2}{8ma^2} \text{ ------------------ (7)}$$

where n = 1, 2, 3…….

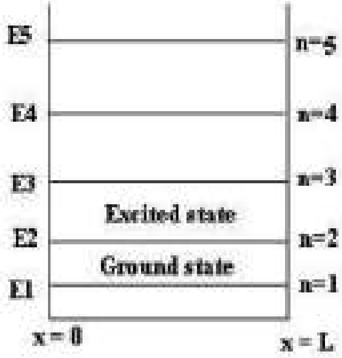
When n = 1 is the lowest energy level given by

$$E_1 = \frac{\mathbf{h}^2}{\mathbf{8m}a^2}$$

This energy is called the zero-point energy. Also,

$E_1 = \frac{h^2}{8ma^2} = E_0$, $E_2 = \frac{4h^2}{8ma^2}$, $E_3 = \frac{9h^2}{8ma^2}$ ……

For each value of 'n' there is an energy level. The possible allowed values of energy obtained from equation (7) i.e., E_1, E_2, E_3 etc are called **Eigen values** and the corresponding wave function ψ_n is called the Eigen function.



The energy corresponding to n = 1 is called ground state energy or zero point energy and the energy levels for n = 2,3,4,5,…., are called excited states.

Inside the well, the particles can have discrete set of values of energy and it is quantized i.e.,

$E_2 = 4E_0$, $E_3 = 9E_0$, $E_4 = 16E_0$, and so on.

Normalization of wave function (evaluation of the value of A):

We have,

$$\psi_n = A \sin\left(\frac{n\pi}{a}\right)x$$

The constant A of this equation can be obtained by applying the normalization condition i.e.,

$$\int_0^a |\psi_n|$$

$$\int \qquad -$$

We know that $\quad -$

Then

$$\int -\left(1 \quad \frac{\quad}{\quad}\right)$$

$$-\left[\int (\qquad \int \frac{\quad}{\quad} dx\right]$$

$$-[x] \qquad - - \left[\frac{\quad}{\quad}\right]$$

$$— \quad —\left[——\right] \quad 0 \Rightarrow S$$

$$— \quad \Rightarrow A \quad \sqrt{-} \ldots\ldots\ldots (8)$$

$$\sqrt{-} \quad —$$

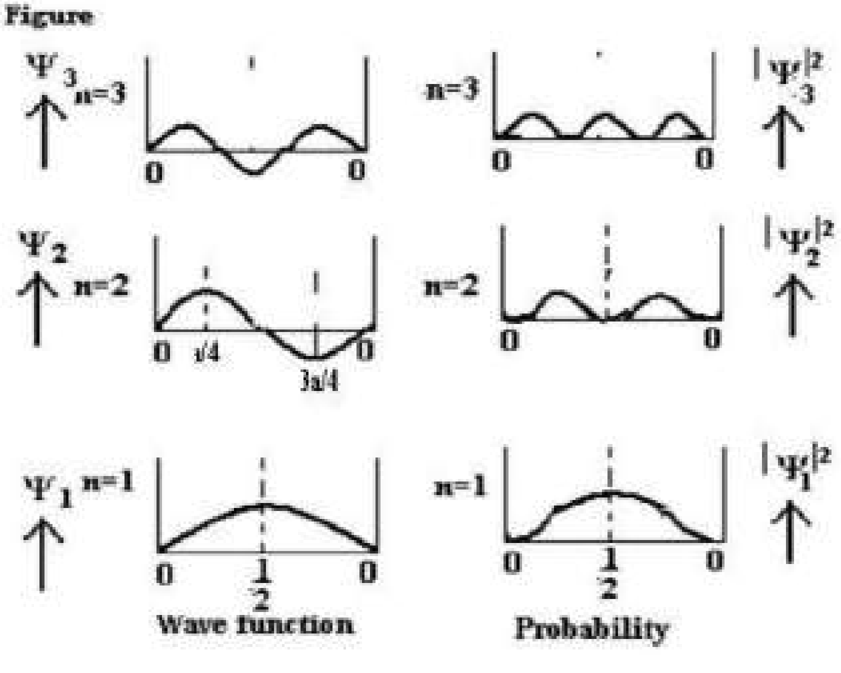
The normalized wave function of the particle is

$$\sqrt{-} \quad —$$

WAVE FUNCTIONS, PROBABILITY DENSITIES AND ENERGY EIGEN VALUES FOR A PARTICLE IN A BOX:

The first three energy levels (Eigen Values) E_1, E_2, E_3 and their corresponding wave function is ψ_1, ψ_2, ψ_3 and probability densities corresponding to n = 1,2, & 3 respectively.

Figure



Case (1)

For n = 1, the eigen function is

$$\psi_1 = A \sin\left(\frac{\pi}{a}\right)x \text{ and } \psi_1 \text{ is maximum at } x = a/2.$$

Thus a plot of ψ_1 versus x will be as shown in figure and a plot of probability density $|\psi_1|^2$ versus x is as shown in figure. The probability of finding the particle is zero both at x = 0 and x = a. It is maximum at x = a/2. This means that in the ground state, the probability of finding the particle is maximum, at the center of the box.

Also, the ground state energy, $E_1 = \frac{h^2}{8ma^2} = E_0$

Case (2)

At n=2, the eigen function for the first excited state is

$\psi_2 = A \sin \frac{2\pi}{a} x$

$\psi_2 = 0$ at x = 0, a/2 and a.

ψ_2 is maximum at the positions x = a/4 and 3a/4.

The plot of ψ_2 versus x and the probability density $|\psi_2|^2$ versus x is as shown in fig.

Since $|\psi_2|^2 = 0$ at x = 0, a/2 and a, this means in the first excited state the particle cannot be observed either at the walls or at the center.

Here, the energy of the first excited state is $E_2 = \frac{4h^2}{8ma^2} = 4E_0$

Case (3)

At n=3 ; the second excited state, the eigen function for the second excited state is given by

$\psi_3 = A \sin \frac{3\pi}{a} x$, also, $\psi_3 = 0$ for x = 0, a/3, 2a/3 and a.

ψ_3 has maximum value for x = a/6, a/2 and 5a/6.

A plot of ψ_3 versus x and probability density $|\psi_3|^2$ versus x is as shown in fig. For each value of n there is an energy level as obtained from equation (1).

Here, $E_3 = \frac{9h^2}{8ma^2} = 9E_0$

Hence, the energy levels are discrete. Each energy value of E_n is called Energy Eigen value. The Eigen value for the corresponding wave function ψ_n is called the Eigen function. The energy values that are permissible in any system are obtained **from the solution of the Schrodinger's** equation and are called Eigen Values.

LASER

LASER: Light Amplification by Stimulated Emission of Radiation.

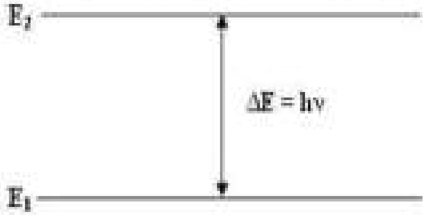
- It was invented by American Scientist Maiman in the year 1960.
- Today there are about hundred different kinds of lasers.

Characteristics of Laser beam

The following important properties of laser make it different from other ordinary source of light.

1) **Laser is highly monochromatic.**
   The laser beam is emitted in a very narrow frequency band.
2) **Laser light is spatially coherent.**
   The laser is highly coherent due to stimulated emission of radiation.
3) **Laser light extremely high directionality or uni-directionality.**
   The laser beam has very small divergence due to the resonant cavity. Hence light intensity does not decrease as fast with distance as it does in ordinary source of light.
4) **The laser beam is extremely bright or intense.**
   Light from laser is much brighter than other ordinary sources of light.

Principle and Production of Laser:



Radiation interacts with matter under appropriate conditions. The interaction leads to an abrupt transition of the Quantum system such as an atom or molecule from one energy state to another. If the transition is from a higher state to a lower one, the system gives out a part of its energy and if the transition is in reverse direction, then it absorbs the incident energy.

In order to understand the manner in which radiation can interact with matter, consider two energy states E_1 and E_2 of a system. If the energy difference between the two energy levels is **ΔE,**

Then $\Delta E = E_2 - E_1$

Max planck suggested that if an electromagnetic radiation of frequency 'ν' with value

$\nu = \frac{}{} = \frac{\quad_2 \quad\quad _1}{}$ -------(1)

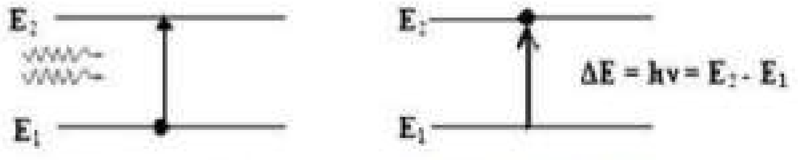
- Induced Absorption
- Spontaneous Emission
- Stimulated Emission

Induced Absorption:

It is a process in which an atom in the ground state undergoes transition to the higher energy state by absorbing an incident photon. This process can be represented as;

Atom + Photon → Atom *

Where Atom* indicates an excited atom

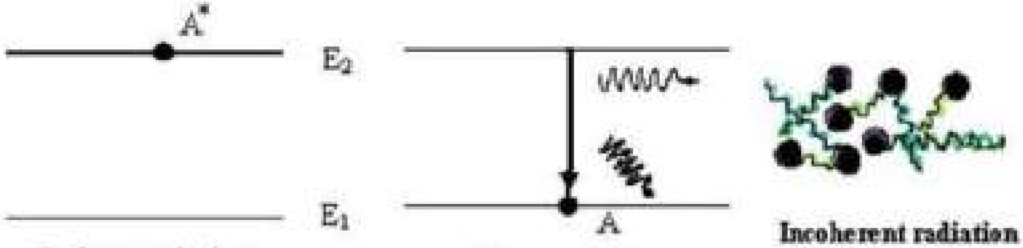


Before Absorption **After Absorption**

Spontaneous Emission:

It is a process in which an atom in the excited state undergoes transition to the ground state by emitting a photon without any aid of external agency. As shown in the figure, consider an atom in the excited state E_2. It makes a transition to the ground state E_1 by the emission of a photon of energy **hν**. It may be represented as

Atom* → Atom + Photon

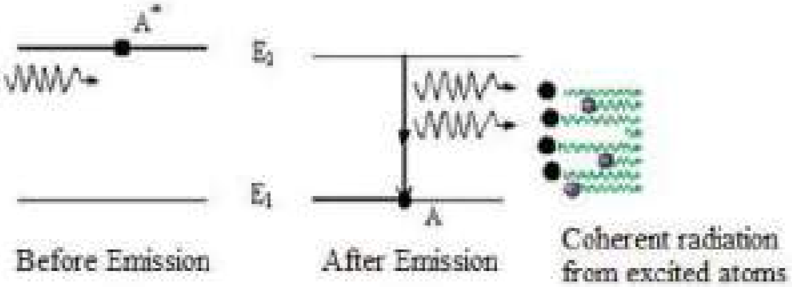


Before emission After emission **Incoherent radiation from excited atoms**

Stimulated Emission:

It is a process in which an atom in the excited state undergoes transition to the ground state by the influence of a passing photon. During this process a stimulated photon is emitted along with the incident photon and these photons are found to be coherent.

Atom * + Photon → atom + 2 Photons (Photon + Photon)



Einstein's Coefficients:

(Expression for energy density of photons in terms of Einstein's Coefficients under thermal equilibrium condition)

* Consider two energy states E_1 and E_2.

* Let E_1 be the lower energy state and E_2 be the higher energy state.

* Let N_1 be the number of atoms per unit volume in the energy state E_1 and

N_2 be the number of atoms per unit volume in the energy state E_2.

* Let be the energy density of radiations.

1. Induced Absorption:

In this case, an atom in the lower energy state E_1 undergoes transition to the higher energy state E_2 by absorbing a photon.

The number of such absorptions per unit time per unit volume is called Rate of induced absorption.

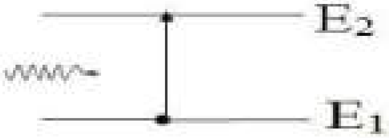
Rate of induced absorption **N_1**

$$= B_{12} N_1 E_\nu \text{ .... } (1)$$

Where N_1 is number of atoms in the state E_1,

is the energy density in frequency range ν and $\nu + d\nu$ and

B_{12} is called Einstein coefficient of induced absorption.

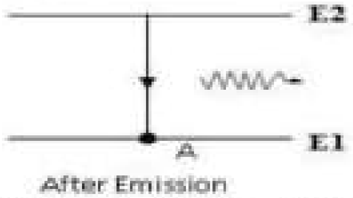


2. Spontaneous Emission:

In this case, an atom in the higher energy state E_2 undergoes transition to the lower energy state E_1 by emitting a photon without any aid of external agency. The number of such spontaneous emissions per unit volume per unit time is called Rate of spontaneous emission.

Rate of spontaneous emission $\propto N_2$

$$= A_{21} N_2 \text{ .... } (2)$$

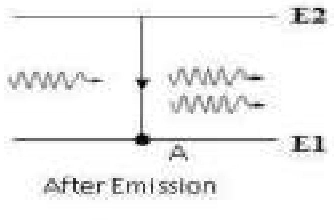


Where, A_{21} is called Einstein's coefficient of spontaneous emission.

3. Stimulated Emission:

In this case, an atom in the higher energy state E_2 undergoes transition to the lower energy state E_1 under the influence of passing photon.

During this process a stimulated photon is emitted along with the incident photon.



The number of such stimulated emissions per unit time per unit volume in called the Rate of stimulated emission.

Rate of stimulated emission $\propto N_2$

$$= B_{21} N_2 \text{ ------ } (3)$$

Where, B_{21} is called the Einstein coefficient of stimulated emission

At thermal equilibrium, the number of upward transitions must be equal to the number of downward transitions.

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$B_{12} N_1 \boldsymbol{E_\nu} = A_{21} N_2 + B_{21} N_2$$

$$(B_{12} N_1 - B_{21} N_2) = A_{21} N_2$$

$$= \frac{\quad}{\quad}$$

$$\frac{\quad}{_2\left[\frac{\quad}{\quad} \quad 1\right]}$$

$$= \frac{\quad}{_{21}\left[\frac{\quad}{\quad} \quad 1\right]} \quad \ldots\ldots\ldots\ldots(4)$$

By Boltzmann law, we have

$$\frac{\quad}{\quad} = e^{\left(\frac{\quad}{KT}\right)} = e^{\frac{h\nu}{\quad}}$$

(4) becomes,

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{KT}} - 1}\right] \quad \ldots\ldots\ldots\ldots(5)$$

According to Planck's law, the equation for energy density of radiation at given temperature, is

$$E_\nu = \frac{8\pi h\nu^3}{C^3}\left[\frac{1}{e^{\frac{h\nu}{KT}}-1}\right] \quad \ldots\ldots\ldots\ldots(6)$$

Comparing equation (5) and (6), we get

$$\frac{\mathbf{A_{21}}}{\mathbf{B_{21}}} = \frac{\mathbf{8\pi h\nu^3}}{\mathbf{c^3}} \textbf{ and } \frac{\mathbf{B_{12}}}{\mathbf{B_{21}}} = \mathbf{1} \textbf{ or } \mathbf{B_{12} = B_{21}}$$

This means that the probability of induced absorption is equal to the probability of stimulated emission. By neglecting the subscripts, A_{21} and B_{21} can be represented as A and B respectively i.e., A_{21} = A and B_{21} = B.

Then at thermal equilibrium, the equation for energy density is

$$\boxed{E_{\nu} = \frac{A}{B\left[e^{\frac{h\nu}{KT}} - 1\right]}}$$

Energy states of atoms:

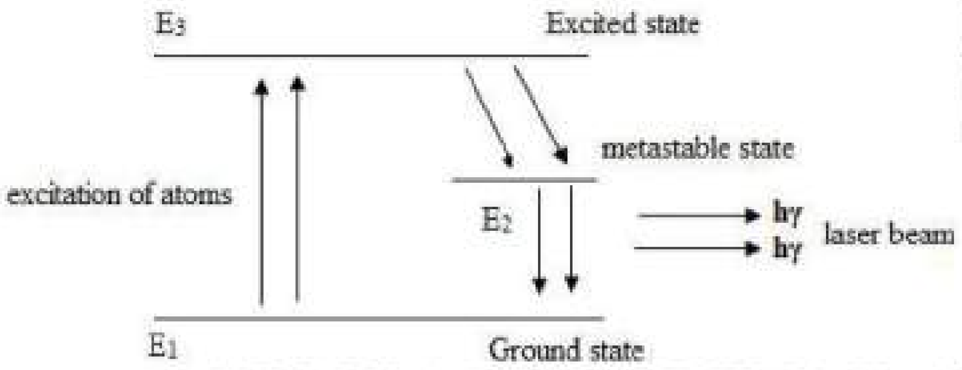
Ground state: It is the lowest possible energy state of an atom which is the most stable state. Atoms can remain in this state for unlimited time.

Excited state: These are the possible energy states of an atom which are higher than the ground state. Atoms remain in these energy states for a very short time called the lifetime typically of the order of 10^{-8} s to 10^{-9} s.

Metastable State: These are excited states of an atom with relatively large lifetime of the order of 10^{-3} s.

CONDITION FOR LASER ACTION:

(Population inversion and metastable state)



"Population inversion is the state of a system at which the population of a particular higher energy state is more than that of a lower energy state". To achieve population inversion a special kind of excited state called metastable state is used and it can be explained as follows.

Atoms in the ground state undergo transition to the higher energy state E_3 by absorbing incident photons. Since E_3 state is ordinary excited state, atoms in the E_3 **state don't stay** over a long time, as a result the atoms immediately undergoes spontaneous downward transitions to the E_2 state. Since E_2 is metastable state, atoms in the E_2 state stay over a long duration of about 10^{-2} to 10^{-3} seconds. Under these conditions a stage will be reached where the population in E_2 overtakes that of E_1. This condition is called population inversion.

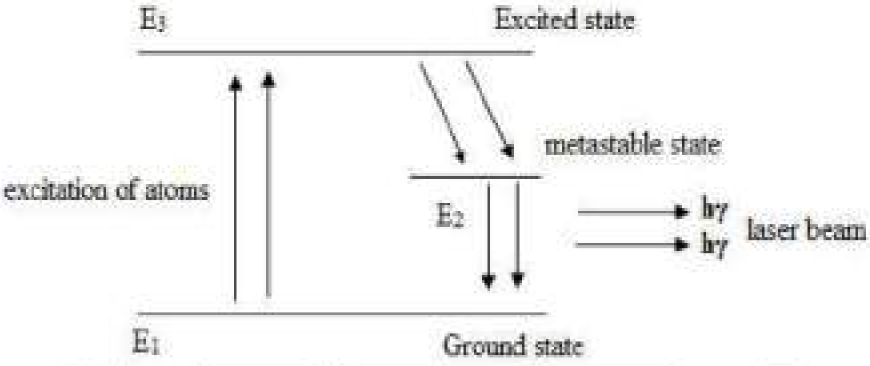
REQUISITES OF A LASER SYSTEM:

There are three requisites of laser systems.
1. An excitation source for pumping action
2. An Active medium to achieve population Inversion
3. An Optical resonant cavity or laser cavity

1. **An Excitation source for pumping action:** The process of supplying energy to the medium to excite an atom from lower energy state to a higher energy state is called pumping.

   Energy can be supplied to atoms in different forms like optical pumping, electrical pumping and chemical pumping.

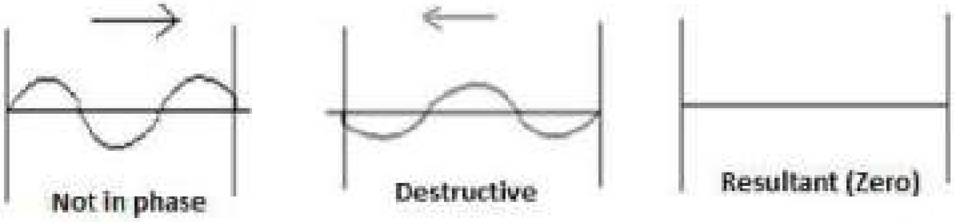
2. **An active medium to achieve Population Inversion:**

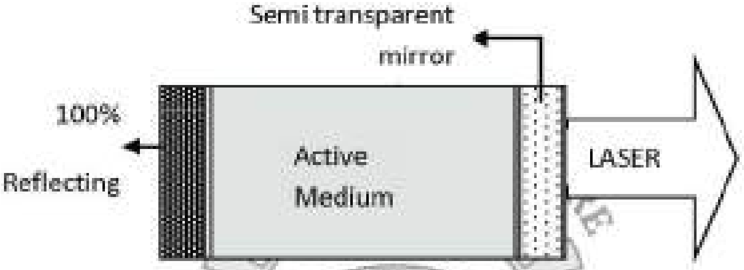


Active medium refers to the medium in which the laser action takes place. The energy levels of the atoms or molecules which are involved in laser action are identified. Accurate information about the energy levels and their lifetimes helps in identifying the level between which the population inversion can be achieved.

3. **An Optical resonant cavity or laser cavity:**

A laser device consists of an active medium bound between two mirrors. The mirrors reflect the photons to and fro through the active medium. A photon moving in a particular direction represents a light wave moving in the same direction. Thus, the two mirrors along with the active medium form a laser cavity.

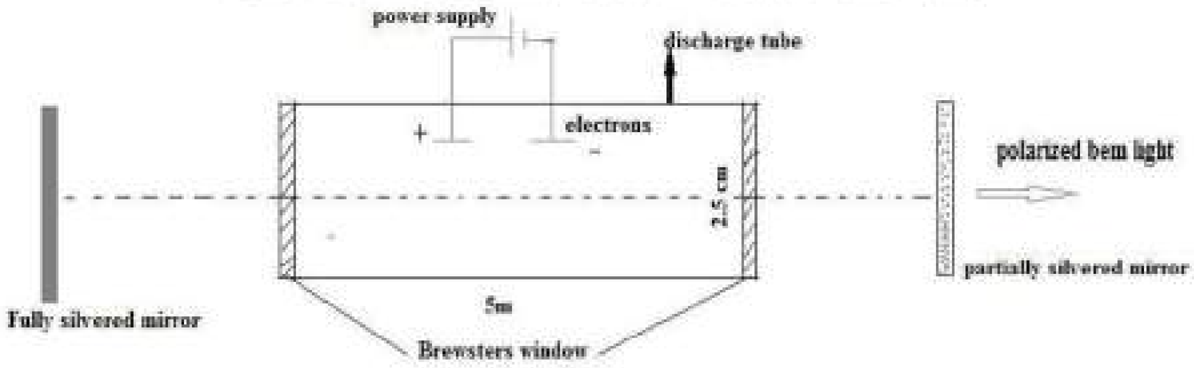




CARBON DIOXIDE LASER [CO_2 LASER]:

CO_2 Lasers belong to the class of molecular gas laser. This operates in the far IR region involving a set of rotational vibrational transitions. CO_2 laser was developed by C.K.N. Patel in the year 1963.

Construction:



1) A CO_2 laser consists of a discharge tube of length 5m and diameter nearly 2.5cm.
2) The tube is filled with a mixture of CO_2 , N_2 and He gases in the ratio of 1:2:3
3) The pressure inside the tube is 6-17 torr.
4) Also water vapour is added to the discharge tube. Water vapour additives help to deoxidize CO to CO_2 in case CO_2 molecules break into CO and O during discharge.
5) The two ends of the tube are sealed with flat Quartz plate which functions as Brewster window(to get the linearly plane polarized light).

6) Two optically plane mirrors are fixed on either sides of the tube, one of the mirrors is fully silvered and can reflect the light 100% and other is partially silvered helps to transmission of laser beam.

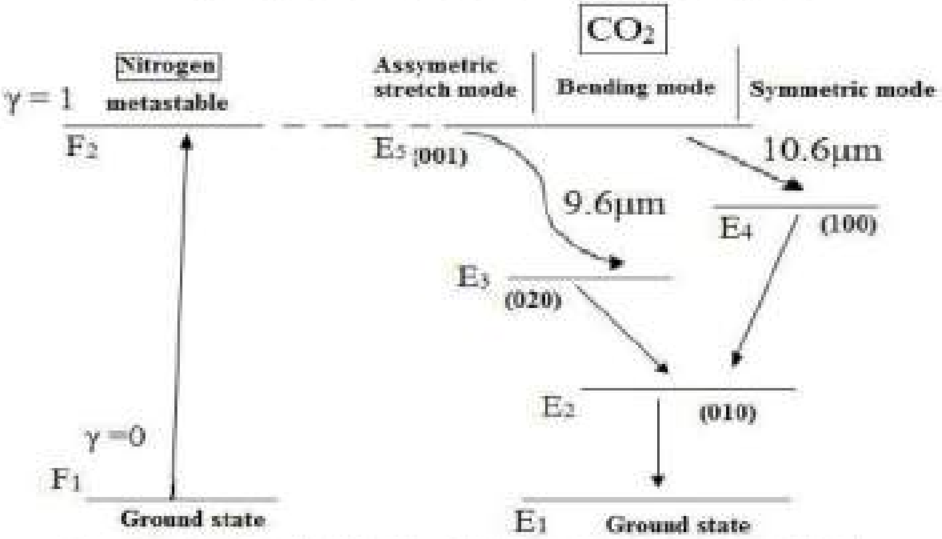
Working:

When the suitable voltage is applied across the two electrodes, a glow discharge of the gases is initiated in the tube. During discharge, electrons gets free from the gas atoms and starts moving towards the positive electrode and begin colliding with N_2 molecule in their path. The collision belong to collision of 1st kind. N_2 molecules are raised to the 1st **vibrational level v = 1 which is a** metastable state. The process is represented as

$$e_1 + N_2 \rightarrow e_2 + N_2^*$$

Where, e_1 and e_2 are energy values of the electrons before and after collision.

N_2 and N_2^* are energy of N_2 **molecules in ground state v = 0 and v =1 state respectively. Hence population inversion is achieved in v =1 of N_2 and** molecules stay for about 10^{-3} to 10^{-2} seconds.



There is a close coincidence in energy of 001 state of CO_2 **and v =1 state of N_2.** Therefore, N_2^* collides with CO_2 at ground state**.** This leads to the transition of CO_2 to 001 state and de-excitation of N_2 to ground state. This type of collision belongs to collision of 2nd kind. 001 state is metastable state for CO_2 molecule. Hence population inversion is achieved in CO_2 molecule in 001 state. This process can be represented by

$$N_2^* + CO_2 \rightarrow CO_2^* + N_2$$

Where, N_2^* and N_2 are energies of N_2 **molecule in v = 1 state** and ground state respectively, and CO_2 and CO_2^* are energies of CO_2 molecule in ground state and 001 state respectively. Let us designate the (010) state, (020) state, (100) state and (001) state as E_2, E_3, E_4, E_5 levels respectively as shown in the figure.

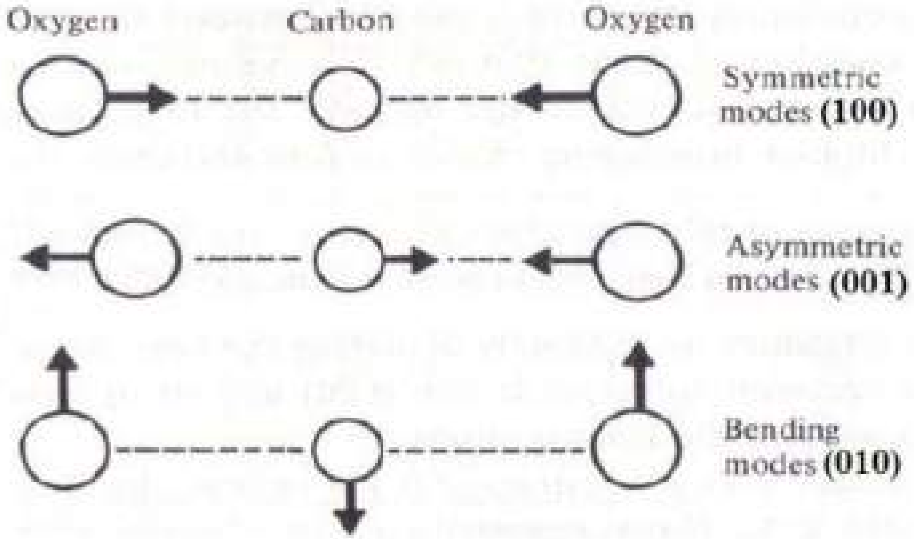
Once population inversion is established in E_5 level the CO_2 molecule undergoes stimulated emission to E_3 and E_4 levels:

(a) Transition from E_5 level to E_4 with a wavelength of 10.6 which is in far IR region.
(b) From E_5 level to E_3 level with a wavelength of 9.6 which is also in far IR region.

Molecule (CO_2) from E_4 level and E_3 level undergoes inelastic collision with unexcited CO_2 molecules and de-excite to E_2 level. Later CO_2 molecule in E_2 level undergoes collision with He and water vapour molecules and came down to the ground state E_1. CO_2 laser has an efficiency of up to 30%.

VIBRATIONAL ENERGY LEVELS OF A CO_2 MOLECULE:

A carbon dioxide molecule has two oxygen atoms between which there is a carbon atom. It has 3 different modes of vibration.



1) **Symmetric stretching mode**: In this mode, oxygen atoms vibrate along the molecular axis, either approaching towards or departing from the carbon atom. The carbon atoms remain stationary. The vibration state is given by (100)
2) **Asymmetric stretching mode**: During the vibration in this mode, all the 3 atoms oscillate along the molecular axis. But, the two oxygen atoms move in one direction while the carbon atom moves in the opposite direction. This vibrational state represented as (001).

3) **Bending mode:** In bending mode, all the 3 atoms oscillate normal to the molecular axis. While vibrating, the two oxygen atoms pull together in one direction as the carbon atom is displaced in the opposite direction. The state is (010).

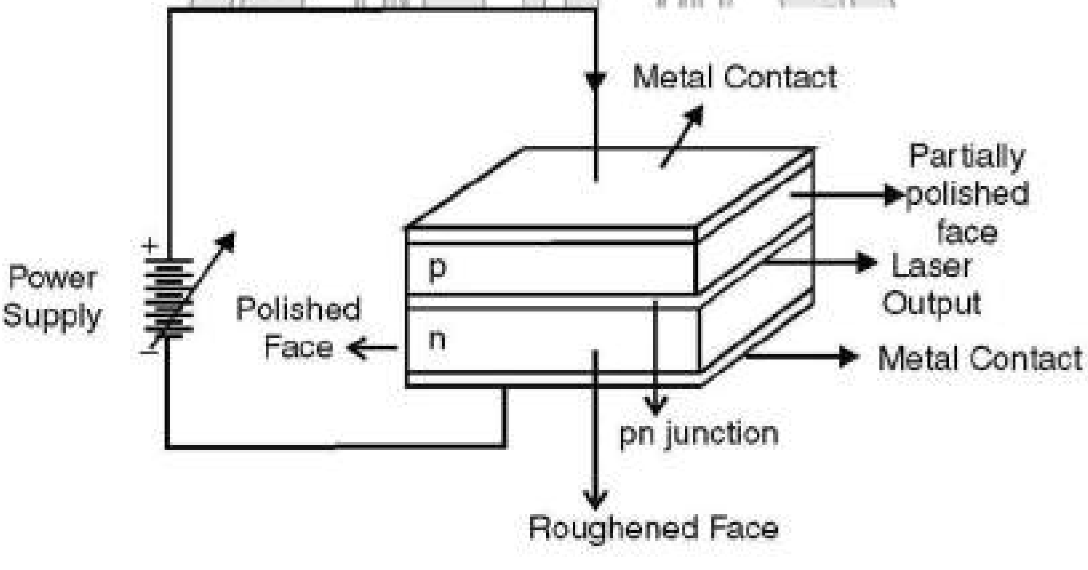
SEMICONDUCTOR LASER (GALLIUM ARSENIDE LASER):

Principle: A Semiconductor diode laser is a specially fabricated p-n junction diode which emits **light when it is forward biased. The 'n'** junction is the active medium. Recombination is the process wherein electrons and holes meet each **and result in the release of heat energy. It's** actually the transition of an electron from conduction band to valence band. Recombination occurs due to forward biasing the system and occurs in the p-n junction (depletion region).Hence it is called as active medium**.** We make use of Direct Band gap and degenerate semiconducting materials

Direct band gap semiconductors are the materials where, there is no loss of energy and the released energy will be in visible region. Semiconductors that are highly doped are called degenerate semiconductors. The p-region is degenerate in holes and the n-region in electrons.

Example GaAs (Gallium Arsenide), CdSe (Cadmium Selenide).

Construction:



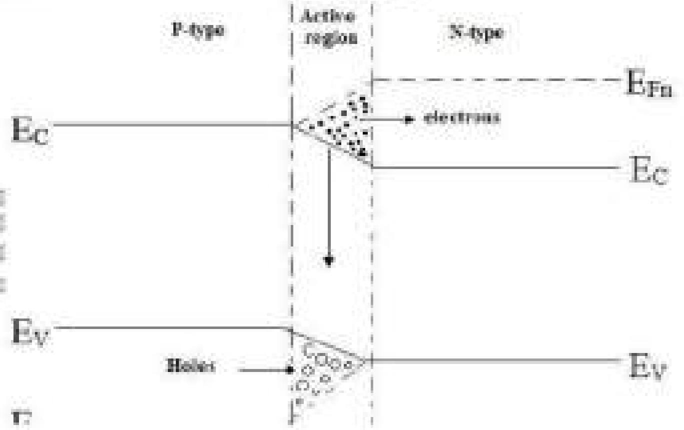
- GaAs diode is a single crystal of Ga and As.
- Consists of heavily doped n and p sections.
- N-section is formed by doping with Tellurium and p-section with Zinc.
- Doping concentration is 10^{17} to 10^{19} dopant atoms/cm^3
- Size of the diode is very small. Sides are 1**mm and junction width is 1μm to 100μm.**

- A pair of parallel planes is polished and these play the role of reflecting mirrors. They provide sufficient reflection to sustain the lasing action.
- Other two sides are roughed surface to suppress the reflections of the photons.
- End surfaces of p-n sections parallel to the plane of junction are provided with the electrodes in order to facilitate application of a forward bias voltage with the help of voltage source.

Working:

- Suitable forward bias voltage is applied to the diode to overcome the potential barrier. Due to forward biasing, more and more electrons are injected into the n-region. This leads to the increase in population of electrons in n-region and population of holes in the p-region. When the current crosses certain value called threshold current, electrons from n-type come to higher energy level of the depletion region and population inversion is attained.
- Once the populations of charge carriers in the depletion region increases, the electrons are made to recombine with the holes in the lower energy level of depletion region.
- At this stage, a photon released by spontaneous emission may trigger stimulated emissions over a large no. of recombination leading to the buildup of laser radiation of high power. Thus, the current flow provides pumping in semiconductor laser.
- The wavelength of emitted light is

$\lambda = \text{—} = \_\_\_\_\_\_\_\_\_\_ = 8874$ Å. The energy gap of GaAs is 1.4 eV.



Applications of semiconductor laser:

1) Used in optical communication
2) Used as reading devices for compact disc players.
3) Semiconductor lasers are used in laser printers.
4) Semiconductor lasers are used in medicine, interferometry and barcode scanners.

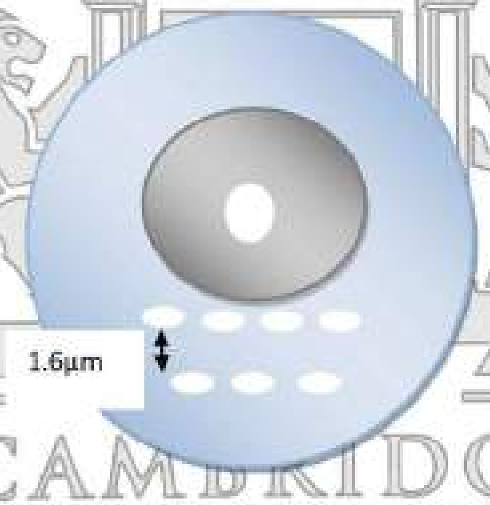
APPLICATIONS OF LASER

Laser range finder

Laser rangefinders have numerous applications such as measuring of rooms and buildings in the construction sector, to determine the depth of snow in inaccessible areas, Cloud base height for atmospheric study, air pollutant distribution, attitude characterization of space debris, trajectory of aircraft, satellites. Laser technology is more cost effective. The laser range finder uses a laser signal is transmitted and returned from a target. The time delay between transmission and receipt of the signal is used to determine the distance to the target based on the speed of light. The receiver consists of reflector, photo detector and amplifier.

Lasers in data storage

In a compact disc, series of microscopic holes known as pits are formed by burning. Laser light is reflected from the disc surface and is detected by photodiodes. The amount of light received by the diodes varies according to the presence or absence of pits.



In a CD, 1s and 0s are recorded in the form of pits along a spiral track on a plastic material with a metal coating. The total length of the track would be around 6 km. Any transition from pit to land or land to pit is read as 1 while the region completely in the land or pit is read as 0. Separation between tracks is 1.6µm in CD and 1.1µm in a DVD (Digital Video Disk). The laser beam is focused on the surface of CD. The reflected beam reaches photo detector and processed. The laser spot should have minimum size. Holographic storage uses entire volume of the recording medium rather than the surface and hence stores large data.

MODULE-5

MATERIAL SCIENCE

QUANTUM FREE ELECTRON THEORY OF METALS

Review of classical free electron theory: (drude – lorentz theory)

Postulates:

1. A metal is assumed to possess a three dimensional array of positive ions with randomly moving free electron gas confined to metallic boundary.
2. These free electron gas is treated as equivalent to gas molecules and they are assumed to obey the laws of kinetic energy of gases. In the absence of any electric field the energy associated with electrons is equal to

$$\text{Kinetic energy} = \frac{3}{2}kT$$

3. The electric current in a metal is due to the drift of electrons in a direction opposite to applied Electric field.

4. The electric field due to all the ions is assumed to be constant.

Drift velocity:

The net displacement in the position of electrons per unit time caused by the application of electric field is known as drift velocity.

$$V_d = \frac{eE\tau}{m}$$

Where e – charge on the electron, E – Electric intensity, τ – Mean collision time.

Mean Collision Time (τ) : It is the average time taken between two consecutive collisions of electrons.

Relaxation time: It is the time taken for the drift velocity to decay to (1/e) times after the removal of electric field. $\tau_r = \dfrac{\tau}{(1-\cos\theta)}$

Mean free path (λ) **:** The average distance traveled by the electrons between two successive collisions.

Expression for the electric current through a metal: Electric current $I = neAV_d$

EXPRESSION FOR ELECTRICAL CONDUCTIVITY (σ):

From Ohms law $J = \sigma E$

$$\therefore \sigma = \frac{J}{E} = \frac{nev_d}{\left(\frac{mv_d}{e\tau}\right)} = \frac{ne^2\tau}{m}$$

Mobility of electrons:

It is net drift velocity acquired per unit electric field applied. Mobility of electrons in Graphene is very much higher than that in Silicon.

$$\mu = \frac{v_d}{E}$$

It can be shown that $\sigma = ne\mu$.

FAILURES OF CLASSICAL FREE ELECTRON THEORY

1. **Prediction of low specific heats for metals:**

Classical free electron theory assumes that conduction electrons are classical particles similar to gas molecules. Hence, they are free to absorb energy in a continuously. Hence metals possessing more electrons must have higher heat content. This resulted in high specific heat given by the expression $C_V = 10^{-4} R$.

This was contradicted by experimental results which showed low specific heat for metals.

$C_V = \frac{3}{2}R$.

2. **Temperature dependence of electrical conductivity:**

From the assumption of kinetic theory of gases

$$\frac{3}{2}kT = \frac{1}{2}mv^2$$

$$v \propto \sqrt{T}$$

Also mean collision time τ is inversely proportional to velocity,

$$\tau \propto \frac{1}{v}$$

$$\tau \propto \frac{1}{\sqrt{T}}$$

$$\therefore \sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto \frac{1}{\sqrt{T}}$$

However experimental studies show that $\sigma \propto \frac{1}{T}$

3. **Dependence of electrical conductivity on electron concentration:**

As per free electron theory, $\sigma \propto n$

The electrical conductivity of Zinc and Cadmium are 1.09 x 10^7 /ohm m and .15 x 10^7 /ohm m respectively which are very much less than that for Copper and Silver for which the values are 5.88x10^7 /ohm m and 6.2 x 10^7 /ohm m. On the contrary, the electron concentration for zinc and cadmium are 13.1x10^{28} /m^3 and 9.28 x10^{28} /m^3 which are much higher than that for Copper and Silver which are 8.45x10^{28} /m^3 and 5.85 x10^{28} /m^3.

These examples indicate that $\sigma \propto n$ does not hold good.

4. Mean free path, mean collision time found from classical theory are incorrect.

QUANTUM FREE ELECTRON THEORY OF METALS

ASSUMPTIONS:

The assumptions of Quantum free electron theory are,

1. The free electrons possess energy and these energy values are quantized. The allowed energy levels are realized in terms of a set of energy levels.

2. The distribution of electrons in various energy levels occurs as per **Pauli's exclusion principle** i.e., single energy level can be occupied by only 2 electrons with opposite spin.

3. The electric potential throughout the lattice is taken to be constant.

4. The attraction and the repulsion between the lattice points and electrons (or) lattice points themselves (or) electrons themselves are ignored.

DENSITY OF STATES

Energy levels of electrons in a solid will look like bands. On a closer look we see that the energy levels in the band are not uniformly distributed. The number of energy levels/unit energy range is explained by density of states.

"Density of states is the number of allowed energy levels in infinitesimally small energy increment dE in the valence band of the material.

Given by, $$\mathbf{g(E)dE} = \frac{-}{h^3}\,{}^{-}\,\mathbf{dE}$$

$$\mathbf{g(E)}\alpha\ {}^{-}$$

FERMI ENERGY

Consider a metal of N atoms and hence there will be N allowed energy levels in each band. The **electrons will occupy energy levels according to Pauli's exclusion principle. When no external** energy is supplied for the electrons, such as thermal energy i.e., at t=0K, we see that levels above valence band will be vacant.

"Fermi energy is the energy corresponding to the highest occupied level at zero degree absolute temperature and the energy level is referred to as the Fermi level (E_F)"

At T=0K, energy levels lying above the Fermi level are empty and those lying below are completely filled.

FERMI FACTOR AND DEPENDANCE OF FERMI FACTOR ON TEMPERATURE AND ENERGY

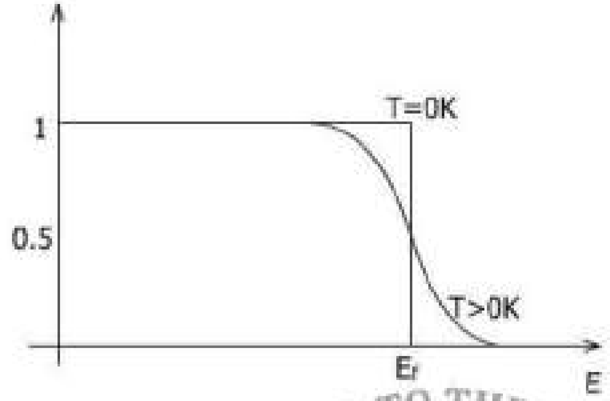
(Probability of occupation of various energy states by elctrons at t=0k and t>0k, on the basis of fermi factor)

Fermi-Dirac Statistics permits the evaluation of probability of finding electrons occupying energy levels in a certain energy range. This is done through a function called Fermi Factor f(E).It is given by,

$$\mathbf{f(E)} = \frac{}{e^{\overline{\quad}}}$$

"Fermi factor is the probability of occupation of a given energy state for a material in thermal **equilibrium"**

Probability of occupation is considered for following cases;



i) Probability of occupation for $E<E_F$ at T=0K

We have, F(E)=

If, $E<E_F$, $E-E_F$ will be negative and T=0,

f(E)=

f(E)=

f(E)=1

- f(E)=1 means, the energy level is certainly occupied. i.e, there is 100% probability that the electrons occupy the energy level below Fermi energy
- All the energy levels below Fermi level are occupied
- $E<E_F$ applies to all the energy levels below E_F

ii) Probability of occupation for $E>E_F$ at T=0K

If $E>E_F$, $(E-E_F)$ will be positive and for T=0,

f(E)=

f(E)=0

- All the energy levels above Fermi level are unoccupied. i.e, 0% probability for the electrons to occupy the energy level above the Fermi level

iii) Probability of occupation at ordinary temperature.

At ordinary temperature, though value of probability remains 1 for $E<<E_F$, it starts decreasing from 1 as the value of E become closer to E_F

For $E=E_F$,

$f(E)=\frac{}{}=\frac{}{}=-$

$f(E)=-$

There is 50% probability for the electrons to occupy the Fermi energy level.

EXPRESSION FOR ELECTRICAL CONDUCTIVITY AS PER QUANTUM FREE ELECTRON THEORY

$$\sigma=\frac{ne}{*}$$

where, m^* → effective mass of the electron
V_F → Fermi velocity
λ → Mean free path
e → charge of the electron
n → electron concentration

SUCCESS OF QUANTUM FREE ELECTRON THEORY

1. **Specific heat (C_V)-** The small value of specific heat (C_V) for the conduction electrons can be explained as follows.

According to quantum theory, only those electrons that occupy energy levels close to E_F can absorb the energy and move to the higher energy states. Number of such electrons absorbing the energy is small and hence C_V is small. It is given by,

$C_V=\left(\frac{}{}\right)RT$

Ex., For E_F=5eV

$\left(\frac{2K}{}\right)\approx10^{-4}$

$C_V=10^{-4}RT$.

This confirms with the experimentally observed values.

2. **Temperature dependence of electrical conductivity.**

The experimental fact that the electrical conductivity σ has dependence on $-$ and not on $\frac{}{}$ is explained here,

We have, $\sigma=\frac{}{}$........(1)

The dependence of σ on T is analyzed as follows,

The free electrons are subjected to scattering due to vibration of ion in the lattice. If ‘r’ is the amplitude of vibration ,it is same in all the directions then the ions can be considered to be present effectively in a circular cross-section of area Πr^2.The value of mean free path λ decreases when Πr^2 increases. i.e, $\lambda \alpha \frac{1}{{}_2}$.........(2)

Now consider the facts,

- Energy of ion α square of the amplitude
  i.e, $E \alpha r^2$........(3)
- Energy of ion α Temperature
  i.e, $E \alpha T$..........(4)
  from eqn (3) & (4) $\rightarrow r^2 \alpha T$..............(5)
  from eqn (2) $\rightarrow \lambda \alpha \frac{}{{}_2}$...............(6)
  from eqn (5) & (6)$\rightarrow \lambda \alpha -$...............(7)
  from (1) & (7)$\rightarrow \sigma \alpha -$

3.Electrical conductivity and electron concentration.

It was not possible to understand why metals such as Aluminium and gallium which have 3 free electrons/atom have lower electrical conductivity than metals such as copper and silver which possess only 1 free electron/atom. Quantum free electron theory could explain why,

We know that , $\sigma \frac{}{}$...............(1)

From(1),it is clear that ‘σ’ depends on both ‘n’ and also on $\left[\frac{}{}\right]$ ratio

The value of ‘n’ for aluminium is 2.13 times higher than that of copper. But the values of $\left[\frac{}{}\right]$ for copper is about 3.73 times higher than that of aluminium.

Therefore, conductivity of copper exceeds that of aluminium.

PHYSICS OF SEMICONDUCTOR

Semiconductors have conductivity in the range intermediate between those of conductors and insulators. The resistivity of semiconductors lie in the range 10^{-6} to 10^{+8} **Ωm.** Elements such as Silicon (Si), Germanium (Ge) , Selenium (Se) and compounds such as Gallium Arsenide (GaAs), Gallium Phosphide (GaP) are some examples of semiconductors.

Electron density in conduction band is given by

$$n_e = 2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}}$$

Hole density in valence band is given by

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

EXPRESSION FOR FERMI LEVEL IN INTRINSIC SEMICONDUCTOR

Electron density in conduction band is given by

$$n_e = 2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}}$$

Hole density in valence band may be obtained from the result

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

For an intrinsic semiconductor, $n_e = n_h$

$$2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}} = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

$$\left(\frac{m_e^*}{m_h^*}\right)^{\frac{3}{2}} = e^{\frac{-E_f + E_v + E_c - E_f}{kT}}$$

$$\frac{3}{2}\ln\left(\frac{m_e^*}{m_h^*}\right) = \frac{-2E_f + E_v + E_c}{kT}$$

$$E_f = \frac{E_v + E_c}{2} - \frac{3}{4}kT\ln\left(\frac{m_e^*}{m_h^*}\right)$$

EXPRESSION FOR CONDUCTIVITY OF SEMICONDUCTING MATERIALS

Carrier concentration in an intrinsic semiconductor:

The number of electrons in the conduction band per unit volume of the material is called electron concentration (N_e) given as,

$$N_e = \frac{-}{} (\pi KT\, m_e^*)^{3/2} \ \overline{\qquad} \ \ldots\ldots\ldots\ldots\ldots (1)$$

Where is the effective mass of the electron

E_F is the Fermi Energy and

E_g is the energy gap.

Similarly, the number of holes in the valence band per unit volume of the material is called hole concentration. (N_h)

$$N_h = \frac{-}{} (\pi KT\, m_h^*)^{3/2} \ \ldots\ldots\ldots\ldots\ldots (2)$$

Where is the effective mass of holes

E_F is the Fermi Energy and

E_g is the energy gap.

T is the absolute temperature and

h is the Planck's constant.

In general, the number of charge carriers per unit volume of the material is called carrier concentration.

The charge carriers, both electrons and holes contribute to the conductivity of the semiconductor.

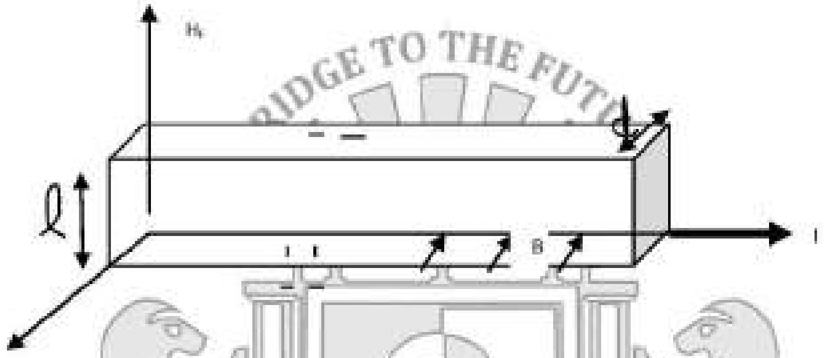
The conductivity of a semiconductor is given by the expression

$$\sigma = e\,[\,(N_e\mu_e) + (N_h\mu_h)\,]$$

where N_e and N_h are the carrier concentrations of electrons and holes respectively.

μ_e and μ_h are mobility of electrons and holes respectively and 'e' is the magnitude of charge of the electron.

HALL EFFECT When a conductor carrying current is placed in magnetic field, an electric field is produced inside the conductor in a direction normal to both current and the magnetic field.



Consider a rectangular slab of an n type semiconductor carrying a current I along + X axis. Magnetic field B is applied along –Z direction. Now according to Fleming's left hand rule, the Lorentz force on the electrons is along +Y axis. As a result the density of electrons increases on the upper side of the material and the lower side becomes relatively positive. This develops a potential V_H-Hall voltage between the two surfaces. Ultimately, a stationary state is obtained in which the current along the X axis vanishes and a field E_y is set up.

Expression for electron concentration

At equilibrium, Lorentz force is equal to force due to electrons

$$BeV_H = -\,e\,E_H$$

$$E_H = BV_H$$

$$\text{But, } E_H = \frac{V_H}{l}$$

$$V_H = l\,E_H = B\,l\,V$$

Current density J

$$\text{Current density } J = n_e ev = \frac{I}{\text{Area}} = \frac{I}{ld} \quad \text{................(1)}$$

$$v = \frac{I}{n_e ld}$$

Hence $V_H = \dfrac{BLI}{n_e l d}$ ....(2)

Electron concentration $n_e = \dfrac{BI}{V_H d}$

Expression for Hall coefficient (R_H)

Hall field is directly proportional to current density (J) and Magnetic field(B).

$E_H \; \alpha \; B$

$E_H \; \alpha \; J$

$E_H = R_H J B$ where R_H is a constant known as Hall coefficient.

$E_H = R_H J B$

$R_H = \dfrac{E_H}{JB} = \dfrac{1}{n_e}$ (From (1) and (2))

Expression for mobility of charge carrier

Mobility $\mu = \dfrac{v}{E}$ ....(1)

Current density $J = \sigma E = n_e e \mu E = n_e e v$ ....(2)

Simplifying (1) and (2)

$$\mu = \sigma R_H$$

Hall effect can be used to

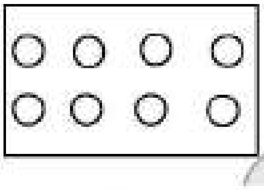
1. Determine the type of semiconductor
2. Calculate carrier concentration, mobility
3. Calculate Magnetic flux density B
4. Determine power in an electromagnetic wave

DIELECTRIC MATERIALS

A **dielectric** (or **dielectric material**) is an electrical insulator that can be polarized by an applied electric field. When a dielectric is placed in an electric field, electric charges do not flow through the material as they do in an electrical conductor but only slightly shift from their average equilibrium positions causing dielectric polarization.

Dipole: Two equal and opposite charges separated by a small distance constitutes a dipole.

Polarization: The separation of effective centre of positive and negative charges in a substance by the application of electric field is known as polarization.



Atoms in a dielectric in the absence of electric field

Atom in a dielectric in presence of electric field

Dipole moment is the product of charge and the separation distance.

$$\mu = ql$$

Atomic polarizability p = αE where α is the polarization constant.

Polarization P= Dipole Moment per unit volume= $\varepsilon_0 E(\varepsilon_r - 1) = N\alpha E_i$

Polar dielectrics: These possess permanent dipole moment .They are permanently polarized in nature.

Ex: Water, KCl, NH_3

Non Polar dielectrics

These are the materials which do not possess permanent dipole moment. They get polarized only in the presence of external electric field. Ex: O_2, N_2, He, Ne

EXPRESSION FOR STATIC DIELECTRIC CONSTANT

(a)In the absence of a dielectric

$$\text{Capacitance } C_{\text{without dielectric}} = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma d}{\varepsilon_o}} = \varepsilon_o \frac{A}{d}$$

(a) In the presence of a dielectric $C_{\text{with dielectric}} = \dfrac{\varepsilon_o \varepsilon_r A}{d}$

$$\text{Static dielectric constant } \varepsilon_r = \frac{C_{\text{with dielectric}}}{C_{\text{without dielectric}}}$$

It is defined as the ratio of capacitance of a capacitor with a dielectric to its capacitance in the absence of a dielectric.

DIFFERENT POLARIZATION MECHANISMS

There are four possible ways in which polarization can in a dielctric when placed in electric field.

1. Electronic polarization
2. Ionic polarization
3. Orientation polarization
4. Space charge polarization

Electronic polarization: These are generally seen in the case of covalent compounds. When a covalent compound is placed in electric field, displacement of electron cloud takes place relative to the nucleus. This displacement creates a dipole which develops dipole moment.

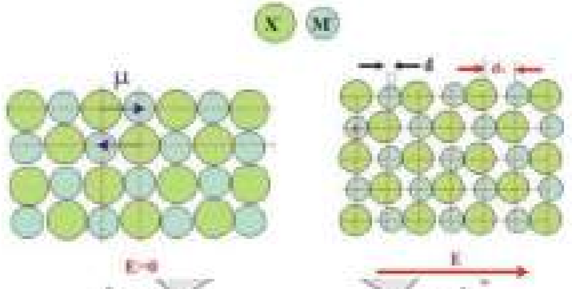
$$\text{Electronic polarizability } \alpha_e = \frac{\varepsilon_o(\varepsilon_r - 1)}{N}$$

Where, N is number of dipoles per unit volume which is independent of temperature.

Ionic polarization:

This is exhibited by ionic compounds.

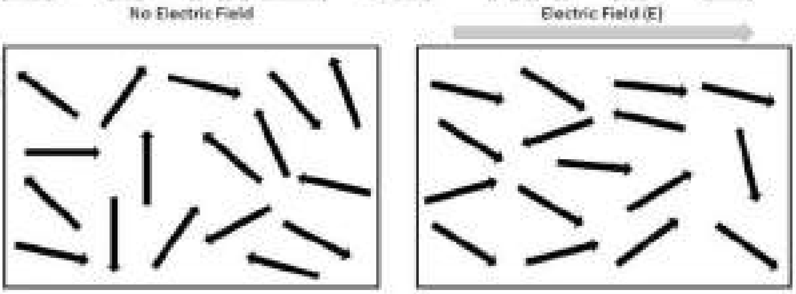
When ionic compounds are kept in an electric field, displacement of positive and negative ions occurs developing a dipole moment.



$$\text{Ionic polarisability } \alpha_i = \frac{\varepsilon_0(\varepsilon_r - 1)}{N_i}$$

Orientation polarization:

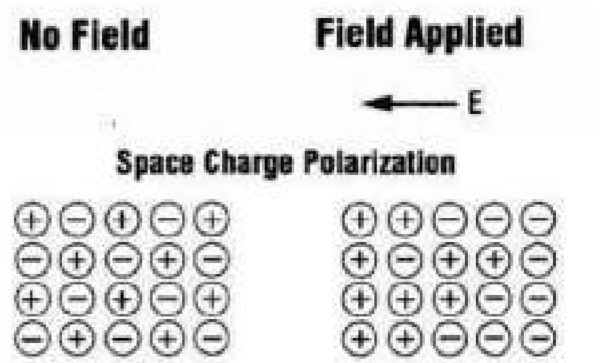
Polar molecules exhibit this mechanism. When polar molecules are kept in an electric field, already existing dipoles tend to align in the direction of applied electric field .This increases the dipole moment.



$$\text{Orientation polarization } \alpha_o = \frac{\mu^2}{kT}$$

Space charge polarization:

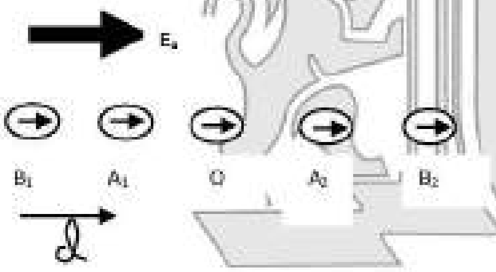
This polarization exists in materials possessing different phases due to difference in temperatures. In such materials charge carriers drift and accommodate in certain regions of higher conductivity (electrodes) causing dipole moment. It occurs in ferrites and semiconductors. Its magnitude is very small compared to other mechanisms.

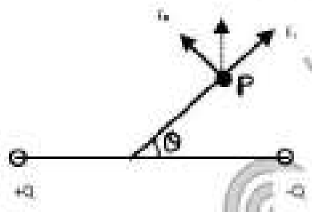


INTERNAL FIELDS IN A DIELECTRIC:

It is the resultant of the applied field and the field produced due to all the dipoles.

Consider a dielectric material kept in an electric field E_a.





The dipole is assumed to be in one dimensional array and is oriented in the same direction.

The electric field at O due to dipole A_1 is given by

$$E_{A1} = \frac{\mu\cos\theta}{2\pi\varepsilon d^3} + \frac{\mu\sin\theta}{4\pi\varepsilon d^3} = \frac{\mu}{2\pi\varepsilon d^3} \text{ as } \theta = 0$$

The electric field at O due to A_2

$E_{A2} = \frac{\mu}{2\pi\varepsilon d^3}$ as all dipoles are oriented in the same direction.

Field at O due to A_1 and A_2 is $E_{A1,A2} = \frac{\mu}{\pi\varepsilon d^3} = E_1$

Similarly field at O due to B_1 and B_2 is $= \frac{\mu}{\pi\varepsilon(2d)^3} = E_2$

The resultant field due n dipoles is given by

$$E_R = E_1 + E_2 + E_3 + E_4 + E_5 + E_6 + \ldots\ldots\ldots\ldots\ldots$$

$$= \frac{\mu}{\pi\varepsilon d^3} + \frac{\mu}{\pi\varepsilon(2d)^3} + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$= \frac{\mu}{\pi\varepsilon d^3}\sum_{n=1}^{\infty}\frac{1}{n^3} = \frac{1.2\mu}{\pi\varepsilon d^3} \qquad \because \sum_{n=1}^{\infty}\frac{1}{n^3} \cong 1.2$$

The internal electric field is $E_i = E_a + \frac{1.2\mu}{\pi\varepsilon d^3}$

$$E_i = E_a + \frac{1.2\mu}{\pi\varepsilon d^3} \qquad \because \mu = \alpha E_a$$

In three dimensional case, $(1/d^3)$ could be replaced by N, the number of atoms per unit volume and (1.2/Π) by a constant γ which depends on the crystal structure.

Hence $$E_i = E_a + \frac{\gamma N\mu}{\varepsilon} = E + \left[\frac{\gamma}{\varepsilon_0}\right]P \qquad \because N\mu = P$$

CLAUSIUS – MOSOTTI RELATION:

This expression relates dielectric constant of an insulator (ε) to the polarization of individual atoms (α) comprising it.

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$$

Where, N is the number of atoms per unit volume
α is the polrisability of the atom
ε_r is the relative permittivity of the medium
ε_0 is the permittivity of free space.

Proof:

If there are N atoms per unit volume, the electric dipole moment per unit volume –known as polarization is given by

$$P = N\alpha E$$

By the definition of polarization P, it can be shown that

$$P = \varepsilon_0 E(\varepsilon_r - 1) = N\alpha E_i$$

$$\varepsilon_0 \varepsilon_r E - \varepsilon_0 E = N\alpha E_i$$

$$\varepsilon_r = 1 + \frac{N\alpha E_i}{\varepsilon_0 E} \text{........................}(1)$$

The internal field at an atom in a cubic structure (γ =1/3) is of the form

$$E_i = E + \frac{p}{3\varepsilon_0} = E + \frac{N\alpha E_i}{3\varepsilon_0}$$

$$\frac{E_i}{E} = \frac{1}{\left[1 - \left(\frac{N\alpha}{3\varepsilon_0}\right)\right]}$$

Substituting for $\frac{E_i}{E}$ in equation (1)

$$\varepsilon_r = 1 + \frac{N\alpha}{\varepsilon_0}\left[\frac{1}{\left(1 - \frac{N\alpha}{3\varepsilon_0}\right)}\right] = \frac{\varepsilon_0\left[1 - \frac{N\alpha}{3\varepsilon_0}\right] + \frac{N\alpha\varepsilon_0}{\varepsilon_0}}{\varepsilon_0\left[1 - \frac{N\alpha}{3\varepsilon_0}\right]} = \frac{1 + \frac{2}{3}\left(\frac{N\alpha}{\varepsilon_0}\right)}{1 - \frac{1}{3}\left[\frac{N\alpha}{\varepsilon_0}\right]}$$

$$\left[\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right] = \frac{\frac{1 + (2/3)\frac{N\varepsilon}{\varepsilon_0}}{1 - (1/3)\frac{N\alpha}{\varepsilon_0}}}{\frac{1 - (2/3)\frac{N\alpha}{\varepsilon_0}}{1 - (1/3)\frac{N\alpha}{\varepsilon_0}} + 2} = \frac{N\alpha}{3\varepsilon_0}$$

SOLID DIELECTRICS

- Jacketing Materials
- Moulding Materials
- Filling Materials

- **Moulding Materials**: These are used for mechanically rigid forms of insulation, for example, insulators, bushings and so on e.g. are ceramics, glass (toughened glass), fiber glass reinforced plastics and epoxy - resins.
- **Jacketing Materials**: Jacketing on a conductor for insulation. Polymers have been found suitable for providing extruded insulating jackets to the conductors. For example, polyethylene (PE), polyvinylchloride (PVC), natural and synthetic (ethylene propylene) rubber are extruded on the conductor in power cables. Polypropylene and paper are used in capacitors and transformers. Mica and fiberglass based polypropylene tapes are used in electrical machines.
- Beside oils, wax - based draining and non - draining impregnating compounds of different types are used to impregnate paper used in power cables, transformers, capacitors, and instrument transformers. Insulating Mechanical Support: In the form of plates, pipes and ledges, insulating supports are required in transformers, circuit breakers and isolators. The products, such as pressboards, hard paper (thin paper laminates), wood (yellow teak) are used in transformers and Bakelite.
- Unlike gaseous and liquid dielectrics, any damage caused to solid dielectrics due to excessive electrical, thermal or mechanical stresses is often irreversible.
- Their thermal and mechanical properties play a very sensitive role since these considerably affect the electrical properties.
- Solid Dielectrics are more exposed to atmosphere, hazards of weather (rain, storm, hail, ice deposits etc.), ultra violet radiation from the sun and pollution (dust, salts etc.)

LIQUID DIELECTRICS

- Insulating oils are used in power and instrument transformers, power cables, circuit breakers, power capacitors, and so on. Liquid dielectrics perform a number of functions simultaneously, namely- insulation between the parts carrying voltage and the grounded container, as in transformers
- Impregnation of insulation provided in thin layers of paper or other materials, as in transformers, cables and capacitors, where oils or impregnating compounds are used
- Cooling action by convection in transformers and oil filled cables through circulation
- Filling up of the voids to form an electrically stronger integral part of a composite dielectric
- Arc extinction in circuit breakers
- Possess a very high electric strength and their viscosity and permittivity vary in a wide range.

CLASSIFICATION OF LIQUID DIELECTRICS

- Organic and Inorganic.
- Organic dielectrics are basically chemical compounds containing carbon. Among the main natural insulating materials of this type are petroleum products and mineral oils, insulating materials are asphalt, vegetable oils, wax, natural resins, wood. A large number of synthetic organic insulating materials are also produced. These are nothing but substitutes of hydrocarbons in gaseous or liquid forms. In gaseous forms are fluorinated and chlorinated carbon compounds. Their liquid forms are chlorinated diphenyles, besides some non-chlorinated synthetic hydrocarbons. The chloro-diphenyles, although possessing some special properties, are not widely used because they are unsafe for humans and very costly.
- Polyisobutylene offers better dielectric and thermal properties than mineral oils for its application in power cables and capacitors, but it is many times more expensive.
- Silicon oils are top grade, halogen free synthetic insulating liquids. They have excellent stable properties, but because of being costly, have so far found limited application for special purposes in power apparatus.
- Among inorganic liquid insulating materials, highly purified water, liquid nitrogen, oxygen, argon, sulfur hexafluoride, helium etc. have been investigated for possible use as dielectrics.
- Liquefied gases, having high electric strength, are more frequently used in cryogenic applications. Water and water mixtures are being actively investigated for use as dielectrics in pulse power capacitors and pulsed power modulators, and so on, because of their high relative permittivity, low cost, easy handling and disposal.

GASEOUS DIELECTRICS

By applying a sensible electrical field, the **dielectric gases** can be polarized. Vacuum, Solids, Liquids and Gases can be a dielectric material. A **dielectric gas** is also called as an insulating gas. It is a dielectric material in gaseous state which can prevent electrical discharge. Dry air, sulfur hexafluoride (SF_6) etc. are the examples of gaseous dielectric materials. Gaseous dielectrics are not practically free of electrically charged particles. When electric field is applied to a gas, the free electrons are formed. A few gases such as SF_6 are strongly attached (the electrons are powerfully attached to the molecule), some are weakly attached for e.g., oxygen and some are not at all attached for e.g. N_2. **Examples of dielectric gases** are Ammonia, Air, Carbon dioxide, sulfur hexafluoride (SF_6), Carbon Monoxide, Nitrogen, Hydrogen etc. The moisture content in dielectric gases may alter the properties to be a good dielectric.

Breakdown in Gases

When subjected to high voltages, gases undergo ionization producing free electrons and begin to conduct.

Properties of Dielectric Gases

The preferred properties of an excellent gaseous dielectric material are as follows

- Utmost dielectric strength.
- Fine heat transfer.
- Incombustible.
- Chemical idleness against the construction material used.
- Inertness.
- Environmentally non-poisonous.
- Small temperature of condensation.
- High thermal constancy.
- Acquirable at low cost

Applications: It is used in transformer, radar waveguides, circuit breakers, Switchgears, high voltage switching, coolants. They are usually used in high voltage application.

APPLICATIONS OF DIELECTRICS IN TRANSFORMERS

In the case of insulating oil, the ability of the oil (acting as a dielectric) to oppose the flow of a current is determined by tests which show its dielectric strength. Specific resistance that a dielectric offers; under moderate conditions of voltage. The ability to prevent arcing between two electrodes at high electric potentials, or to prevent arcing between the electric source and ground. Petroleum oil is an excellent dielectric and it is used extensively as a hydraulic medium for equipment used around electrical apparatus, such as in servicing high voltage electric equipment, lines, or transmission media. Oil is also used in switchgears, transformers and other such electrical equipment designed to be insulated with a liquid. The dielectric strength of insulating dielectric-transformer oil is evaluated in terms of its breakdown voltage under a standard set of conditions.

In liquid filled transformers, dielectric fluid is used to cool the windings and provide optimal performance. From the bottom of the tank where the dielectric fluid is at its lowest or "bottom" temperature, the fluid flows vertically up the winding ducts and is heated by the windings. At the top of the tank, where the fluid is at its highest or "top oil temperature", it exits the main tank and enters a series of radiators or cooling fins. It then flows downward through the radiators, where it is cooled, and reenters the main tank at the bottom. In self-cooled transformers this cycle is governed naturally by convection. Natural convection can also be assisted by a series of fans directing air against the radiators increasing the rate of heat transfer and subsequent rate of cooling in the windings. In some large power transformers it is also possible to have a level of forced oil circulation where a pump assists in the circulation of the fluid. This generally provides a lower top oil temperature and more uniform temperatures within the windings. Mixtures of polychlorinated biphenyls (PCBs) used as a transformer dielectric fluid because of their non-flammable nature and chemical stability.